III SEMESTER B.TECH MAKEUP EXAMINATION , January, 2022

SUBJECT: LINEAR ALGEBRA and LOGIC (MAT 2163)

REVISED CREDIT SYSTEM

Time: 3 Hours Date: 27/1/2023 MAX. MARKS: 50

Instructions to Candidates

- Answer **ALL** the questions.

Q.No	Question	Marks	CO	BL
1A	Determine whether the following system is consistent. If so, compute its general solution and identify the free and basic variables $$x_1 - x_2 - 3x_3 + x_4 = 0$$ $$-2x_1 + x_2 + 5x_3 = -4$$ $$4x_1 - 2x_2 - 10x_3 + x_4 = 5$$	4	1	2,3
1B	Suppose that $T: R^2 \longrightarrow R^2$ is a linear transformation such that $T\left(\begin{bmatrix}1\\-2\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}$ and $T\left(\begin{bmatrix}-1\\3\end{bmatrix}\right) = \begin{bmatrix}3\\0\end{bmatrix}$. Determine $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right)$ for any $\begin{bmatrix}x_1\\x_2\end{bmatrix}$ $in\ R^2$.	3	3	3,4
1C	Find the dimension of the column space of matrix. Mention the basis for the ColA. A= $\begin{bmatrix}1 & 2 & -1 & 2\\-1 & -2 & 1 & 2\\2 & 4 & -3 & 2\\-3 & -6 & 2 & 0\end{bmatrix}$.	3	1	2,3
2A	Solve the system by Cholesky's method $$x + 2y + 3z = 5$$ $$2x + 8y + 22z = 6$$ $$3x + 22y + 82z = -10$$	4	3	4,5
2B	Find a change of variable x = Py that removes the cross-product term from the quadratic form $5{x_1}^2 - 4x_1x_2 + 5{x_2}^2$	3	3	3
2C	Show that the propositional formula $(\neg p\wedge(\neg q\wedge r))\vee(q\wedge r)\vee(p\wedge r) \Leftrightarrow r$.	3	4	2
3A	Find the spectral decomposition of the matrix $A = \begin{bmatrix}1 & 1 & 5\\1 & 5 & 1\\5 & 1 & 1\end{bmatrix}$.	5	3	3,4

3B	Prove that the set $\left\{\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \in R^3 : 2u_1 + 5u_2 - 4u_3 = 0\right\}$ is a subspace of R^3.	3	1	2
3C	Show that $(\forall x)[H(x) \rightarrow M(x)] \wedge H(s) \Rightarrow M(s)$.	2	5	3
4A	Let $b_1 = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}, b_2 = \begin{bmatrix} 0 \\ 4 \\ 2 \end{bmatrix}, b_3 = \begin{bmatrix} 5 \\ 2 \\ 4 \end{bmatrix}$ where the set $B = \{b_1, b_2, b_3\}$ is a basis for R^3. Determine whether $y = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ is an affine combination of points in B.	4	2	2,3
4B	Find the matrices V and Σ in the singular value decomposition of matrix $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$	3	3	3,4
4C	Prove that a parallelogram is a rhombus if and only if the diagonals are perpendicular to each other using the concept of inner product space.	3	2	3,4
5A	Consider the following system of differential equations $$y_1' = -5y_1 - 4y_2$$ $$y_2' = 8y_1 + 7y_2$$ a) Find the general solution of the system b) Find the particular solution of the system that satisfies the initial conditions $y_1(0) = 1$ and $y_2(0) = 4$	4	4	2,3,4
5B	Show that $r \wedge (p \vee q)$ is a valid conclusion from the premises $p \vee q, q \rightarrow r, p \rightarrow m$ and $\neg m$.	3	4	3,4
5C	Apply Gram Schmidt process to construct an orthonormal basis for R^3 using the set $\{(1,-1,0), (2,-1,-2), (1,-1,-2)\}$.	3	2	2,3