III SEMESTER B.TECH SEMESTER EXAMINATION , December, 2022

SUBJECT: LINEAR ALGEBRA and LOGIC (MAT 2163)

REVISED CREDIT SYSTEM

Time: 3 Hours | Date: 22/12/2022 | MAX. MARKS: 50

Instructions to Candidates

- Answer **ALL** the questions.

Q.No	Question	Marks	CO	BL
1A	Determine whether the following system is consistent. If so, compute its general solution and identify the free and basic variables $$x_1 - x_2 - 3x_3 + x_4 = -2$$ $$-2x_1 + 2x_2 + 6x_3 = -6$$ $$3x_1 - 2x_2 - 8x_3 + 3x_4 = -7$$	4	1	2,3
1B	Find the change of variable $x = Py$ that transforms the quadratic form $Q(x): x_1{}^2 - 5x_2{}^2 - 8x_1x_2$ *into* $y^T Dy$. Classify the quadratic form.	3	3	3,4
1C	Find the basis for the null space of a matrix $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ -8 & 3 & 5 & -6 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & -3 & 2 \end{bmatrix}$.	3	1	2,3
2A	The input-output matrix for an economy producing transportation, food and oil is given by $$\begin{array}{ccc} Trans & Food & Oil \end{array}$$ $$\begin{bmatrix} 0.2 & 0.20 & 0.3 \\ 0.4 & 0.30 & 0.1 \\ 0.2 & 0.35 & 0.3 \end{bmatrix} \begin{array}{c} Trans \\ Food \\ Oil \end{array}$$ (a) What is the net production corresponding to a gross production of \$40 million of transportation, \$30 million of food, and \$35 million of oil? (b) What gross production is required to satisfy exactly a demand for \$32 million of transportation, \$48 million of food, and \$24 million of oil?	4	1	4,5
2B	Suppose a particle is moving in a planar force field and its position vector X(t) satisfies $x'(t) = Ax(t)$ and $x(0) = x_0$ where $A = \begin{bmatrix} 4 & -5 \\ -2 & 1 \end{bmatrix}$ *and* $x_0 = \begin{bmatrix} 2.9 \\ 2.6 \end{bmatrix}$. Solve this initial value problem for $t \geq 0$.	3	3	3

2C	Show that the propositional formula $p \to (q \to r)$ logically equivalent to $(p \wedge q) \to r$ using truth table or laws of logic.	**3**	**4**	**2**
3A	Find the singular value decomposition of $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$	**0.5**		
3B	Find a basis for a subspace $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in R^3 : x_1 - 3x_2 + 5x_3 = 0 \right\}$. Show that $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} \right\}$ is basis for the subspace V.	**0.5**	**1**	3
3C	Suppose that $T: R^2 \to R^3$ is a linear transformation such that $T\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$ and $T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$. Determine $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$.	**0.5**	**1**	**3**
4A	Let $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 5 \end{bmatrix}, v_4 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$. If possible, write y as an affine combination of v_1, v_2, v_3 and v_4.	**0.5**	**2**	**2,3**
4B	Find the QR factorization of the matrix $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix}$	**3**	**3**	**3,4**
4C	Let V be an inner product space. For a unit vector u and any vector $v \in V$, the projection $P_u(v)$ of v onto the line is defined as $P_u(v) = < v, u > u$. Prove that $d(P_u(v), v) \leq d(\alpha u, v)$ for any α∈ R	**3**	**2**	**3,4**
5A	Show that the following statements constitute a valid argument using laws of predicate calculus. "If there was a football game, then traveling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no football game".	**4**	**4**	**2,3,4**
5B	Show that$(\exists x)M(x)$ follows logically from the premises $(\forall x)[H(x) \to M(x)]$ and $(\exists x)H(x)$.	**3**	**5**	**3,4**
5C	Let W be the subspace of R^5 spanned by $u = \begin{pmatrix} 1 \\ 2 \\ 3 \\ -1 \\ 2 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 4 \\ 7 \\ 2 \\ -1 \end{pmatrix}$. Find a basis for the orthogonal complement of W.	**3**	**2**	**2,3**