Question Paper

Exam Date & Time: 27-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH. (COMPUTER SCIENCE & ENGINEERING / INFORMATION TECHNOLOGY) EXAMINATIONS - JAN/FEB 2023
SUBJECT : MAT 2155 - ENGG. MATHS III
(MAKEUP)

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Find the 27th and 65th permutation of 5 marks 1,2,3,4,5 in both Lexicographical ordering and Fike's ordering. (4)

1B) If 5 men A, B, C, D, E intend to speak at a meeting, in how many orders can they do so without B speaking before A? How many orders are there in which A speaks immediately before B? (3)

1C) Prove that in a distributive lattice, if $b \wedge \bar{c} = 0$ then $b \leq c$. (3)

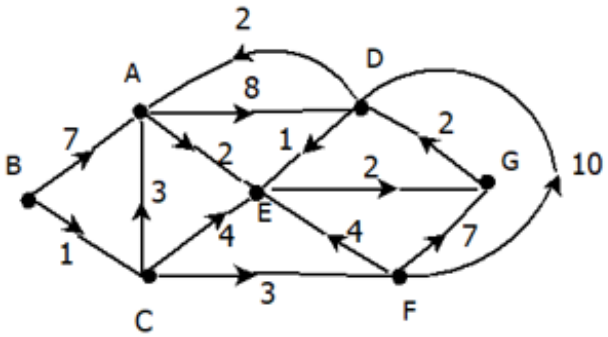
2A) i) Let x and y are elements of a group $(G,*)$ *and* there exist $a \in G$ such that $a * x = a * y$, then prove that $x = y$. (4)

ii) Prove that in a group $(G, *)$, $(x * y)^{-1} = y^{-1} * x^{-1}$ $\forall$ x, y $\in G$.

2B) Prove that Every tree has a centre consisting of either one vertex or two adjacent vertices. (3)

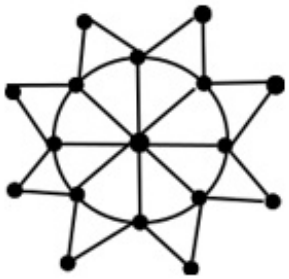
2C) Let (A, $\leq$) be a distributive lattice. Show that $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for some a, then $x = y$. (3)

3A) Using Dijkstra's algorithm, find the shortest path from B to all other vertices for the network given below. (4)



3B) How many ways are there to collect 30$ from 4 children and 6 adults, if each person gives at least 1$, but each child can give at most 5$ and each adult at most 7$. (3)

3C) Define Regular graph, Hamiltonian graph and Eulerian graph. Is the following graph Hamiltonian or Eulerian? Justify your answer. (3)



4A) Using Ferrers diagrams, show that the number of partitions of n into at most k parts is equal to the number of partitions of n with no part larger than k. Hence derive the generating function for the number of partitions of n into at most k parts. (4)

4B) Let $E(x_1, x_2, x_3) = \overline{(\overline{x_1} \vee x_2) \vee (\overline{x_1} \wedge x_3)}$ be a Boolean expression over the two-valued Boolean algebra $\{0, 1\}$. Write $E(x_1, x_2, x_3)$ in both CNF and DNF. (3)

4C) For any a, b, c, d in a lattice (A, $\leq$) if $a \leq b$ and $c \leq d$, then show that $a \vee c \leq b \vee d$ and $a \wedge c \leq b \wedge d$. (3)

5A) Show that $Z_5 = \{0,1,2,3,4\}$ forms a cyclic group under the operation of addition modulo 5. List all the elements of Z_5 that generate it. (4)

5B) Show that $((p \vee q) \wedge \neg[(\neg p) \wedge (\neg q \vee \neg r)]) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology without constructing truth table. (3)

5C) Show that from: (a) $(\forall x)[F(x) \wedge S(x)] \rightarrow \forall y[M(y) \rightarrow W(y)]$
(b) $(\exists y)[M(y) \wedge \neg W(y)]$ the conclusion $(\forall x)[F(x) \rightarrow \neg S(x)]$ follows. (3)

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