Exam Date & Time: 27-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

III Semester BTECH End Semester MAKE UP Examination, JANUARY 2023

MATHEMATICAL FOUNDATIONS FOR DATA SCIENCE - I [MAT 2157]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) Compute the mean and median for the following data

A)

Class	0 -10	10 - 20	20 - 30	30 - 40	40 - 50
frequency	5	15	25	8	7

(3)

B) Person A and B throw alternatively a pair of dice. A wins if he gets sum 6 before B gets sum 7 and B wins if he throws sum 7 before A throws sum 6. If A begin, find their chance of winning? (3)

C) Compute the quartile deviation and standard deviation for the following

X	100-109	110-119	120-129	130-139	140-149	150-159	160-169	170-179
f	15	44	133	150	125	82	35	16

(4)

2) A coin is known to come up heads 3 times as often as tail. This coin is tossed 3 times. Let X be the number of heads that appear. Write out the probability distribution of X and also the cumulative distribution function (cdf) of X. (3)

A)

B) If X, Y, Z are uncorrelated random variables with standard deviations 5, 12, 9 respectively and if U = X+ Y, V = Y + Z, evaluate the correlation coefficient between U and V (3)

C) In a bolt manufacturing factory, there are 3 machines A, B, C. The machine produce 25%, 35%, and 40% respectively of the total product. A bolt is chosen at random from product and found to be defective. It is believed that 5%, 4%, and 2% respectively of the products manufactured by A, B, C are usually defective. What is the probability that the defective bolt selected was manufactured by machine C? (4)

3) X is a Poisson variable and it is found that the probability that $X = 2$ is two third of (3)

probability that $X = 1$. Find the probability that $X = 0$ and $X = 3$. What is the probability that X exceeds 3?

A)

B) R is a resistance to maintain a train at speed V. Find the law of the type $R = a + Vb^2$ connecting V using the following data.

V(miles/hour)	10	20	30	40	50
R(lb/ton)	8	10	15	21	30

(3)

C) Solve
$u_t = u_{xx}, 0 < x < 1, t > 0,\ u(x,0) = 100(x - x^2),\ u(0,t) = 10(t - t^2),\ u(1,t) = 0$
With $h = \frac{1}{4}$ and $\lambda = \frac{1}{4}$ Compute u for four time steps (4)

4) The intelligence quotients (IQ) of a random sample of ten boys are 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ of 90? Test this at 5% level of significance. The threshold value is 2.26. Also find a 95% confidence interval for the mean of the population. (3)

A)

B) Assume that the time of arrival of birds at a particular place on a migratory route, as measured in days from the first day *i. e.,* 1st January of the year, is approximated as a Gaussian (normal) random variable X with mean 200 days and standard deviation 20 days .

What is the probability that the birds arrive after 160 days but on or before 210th day? (3)

What is the probability that the birds will arrive after the 231st day?

P(Z< 0.2)=0.9772, P(Z< 0.5)=0.6915, P(Z< 1.55)=0.9394

C) The table below shows the data collected during the outbreak of small pox. Clearly stating the hypothesis, test the effectiveness of vaccination in preventing small pox at 5% level of significance. The cutoff value at 5% level of significance is 3.84.

	Attacked	Not attacked
Vaccinated	60	440
Not Vaccinated	140	60

(4)

5) Suppose the joint probability density function of (X, Y) is given by
$f(x,y) = \begin{cases} \frac{6}{5}(x + y^2); & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & ; \quad otherwise \end{cases}$ (3)
. Obtain the marginal pdfs of X

and Y

A)

B) Let X and Y be discrete random variables with joint probability mass function $p(x, y)$ given by the following table. Find $Covariance(X, Y)$. Are X and Y independent? (3)

	$X = 2$	$X = 3$	$X = 4$	$X = 5$
$Y = 0$	0.05	0.05	0.15	0.05
$Y = 1$	0.40	0.00	0.00	0.00
$Y = 2$	0.05	0.15	0.10	0.00

C) A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single factor experiment was conducted with three dosage levels and the following results were obtained. (4)

Dosage	Observations			
20g	24	28	37	30
30g	37	44	39	35
40g	42	47	52	38

Is there evidence to indicate that dosage level affects bioactivity? The threshold at 5% level of significance is 4.26

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