Question Paper

Exam Date & Time: 27-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH. (ELECTRONICS AND COMMUNICATION ENGINEERING) EXAMINATIONS - JAN/FEB 2023
SUBJECT : MAT 2152 ENGG. MATHS III
(MAKEUP)

Marks: 50 | **Duration: 180 mins.**

Answer all the questions.

1A) Expand $f(x) = x^2, 0 < x < 2, f(x+2) = f(x)$ as a Fourier Series and hence show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ at x= 0. (4)

1B) Expand $f(x) = 2x - 1, 0 < x < 1$ as a half-range Fourier sine series. (3)

1C) Find the Fourier transform of $f(x) = \begin{cases} 1, & |x| \le a \\ 0, & |x| > a \end{cases}$ and hence evaluate $\int_0^{\infty} \left(\frac{sint}{t}\right)^2 dt.$ (3)

2A) Find the analytic function $\boldsymbol{f(z) = u + iv}$, where (4)

$$\boldsymbol{u = \frac{y}{x^2 + y^2}}$$

2B) Show that $f(z) = e^{-z}$ is analytic. Find $f'(z)$. (3)

2C) Find the Fourier sine transform of the following function, (3)

$$f(x) = e^{(-ax)}, \text{ a} > 0$$

3A) Evaluate $\oint_C \frac{z^2+5}{(z-2)(z-3)} dz$ using residue theorem, C is the circle |z| = 4 (4)

3B) Evaluate the following integral, $\int_C \frac{z^2-z+1}{z-2} dz$ Where C is (3)

$(i)\ |z-1| = 1/2 \quad (ii)\ |z| = 2 \quad (iii) |z| = 1/2$

3C) Find Laurent Series expansion of $\frac{z^2-1}{z^2+5z+6}$, about (i) $|z| < 2$ (ii) $2 < |z| < 3$ (3)

4A) Verify Green's theorem for (4)

$$\oint_C (xy + y^2)\, dx + x^2\, dy,$$

where C is the boundary of the region defined by $y = x$, $y = x^2$

4B) (3)

Find the equation of the tangent plane and normal line to the surface $x^2 + y^2 = z$ at (2, -1, 5).

4C) (a) Show that $\vec{F} = (2xy + z^2)\hat{\imath} + (x^2)\hat{\jmath} + (3xz^2)\hat{k}$ is a conservative force field. (3)

(b) Find its scalar potential.

(c) Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).

5A) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - 2\frac{\partial^2 u}{\partial y^2} = 0$ using the indicated transformations $v = x + y, \; z = 2x - y$. (4)

5B) Assuming the most general solution, Solve the heat equation $u_t = c^2 u_{xx}, 0 \leq x \leq \pi$ to find the temperature distribution at any time, subjected to the condition (3)

$u(0,t) = 0, \; u(\pi, t) = 0$ and

$$u(x,0) = \begin{cases} x & in \; 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & in \; \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

5C) Use Divergence theorem to evaluate $\iint_s F.n\,ds$. Where $F = 4xzi - y^2j + yzk$ and S is the surface of the cube bounded by $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$. (3)

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