Exam Date & Time: 27-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH END SEMESTER MAKE-UP EXAMINATIONS, JAN2023

ENGINEERING MATHEMATICS III [MAT 2151]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions. Section Duration: 180 mins

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1)

A) Solve $y'' - y' - xy = 1$ given that $y(0) = 1$, $y(1) = 5$ taking h = 0.25 using finite difference method. (4)

B) Obtain the solution of the partial differential equation $u_x + u_y = 2(x+y)u$, by the method of separation of variables. (3)

C) If $\vec{A} = xz^3\hat{\imath} - 2x^2yz\,\hat{\jmath} + 2yz^4\hat{k}$ .Find at (1,-1,1) (3)

i. $\nabla.\vec{A}$

ii. $\nabla.(\nabla \times \vec{A})$

2)

A) Verify Green's theorem for the integral $\oint_C (y - sinx)\,dx + cosx\,dy$, where C is the triangle with vertices (0,0), $(\frac{\pi}{2}, 0)$ and $(\frac{\pi}{2}, 1)$. (4)

B) Find the Fourier cosine Transform of $f(x) = \begin{cases} x & : 0 < x < 1 \\ 2-x & : 1 < x < 2 \\ 0 & : x > 2 \end{cases}$ (3)

C) Solve the partial differential equation $u_{xx} + u_{yy} = -81x^2y^2$ by finite difference method with $u(0,y) = u(x,0) = 0$ and $u(1,y) = u(x,1) = 100$ with $h = \frac{1}{3}$. (3)

3)

A) (4)

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi} & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi} & 0 \le x \le \pi \end{cases}$$

Find the Fourier series of Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \ldots \ldots \ldots \ldots \ldots \ldots$

B) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; $0 < x < 1;\ t > 0$ for 2-time steps by Crank-Nicolson's method with $h = \frac{1}{4};\ u(x,0) = 100(x - x^2), u(0,t) = u(1,t) = 0.$ (3)

C) Expand $f(x) = 1 - \frac{x}{l}, 0 < x < l$ as a Fourier Sine series. Also draw the graph of the corresponding periodic extension of f(x). (3)

4) Evaluate $\iint_S \vec{A}.n\,\overrightarrow{dS}$, where $\vec{A} = 18zi - 12j + 3yk$ and S is the part of the plane $2x + 3y + 6z = 12$ in the first octant. (4)

A)

B) Solve $z_{xy} = sinx\ siny$ for which $z_y = -2siny$ when $x = 0$, and $z = 0$ if y is an odd multiple of $\frac{\pi}{2}$. (3)

C) Obtain Fourier series for the function $f(t) = \frac{1}{4}(\pi - t)^2;\ 0 \le t \le 2\pi,\ f(t + 2\pi) = f(t)$. (3)

5) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; 0 < x < 1; t > 0$ for 4-time steps with $h = 0.25$ subject to

A) $u(x,0) = 100(x - x^2),\ \frac{\partial u}{\partial t}(x,0) = 0; u(0,t) = u(1,t) = 0$ (4)

B) Find the directional derivative of $f(x,y,z) = x^2 - y^2 + 2z^2$ at the point $p(1,2,3)$ in the direction of the line PQ where Q is the point (5, 0, 4). Also find the magnitude of the maximum directional derivative. (3)

C) With suitable assumptions, derive one dimensional wave equation. (3)

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