Exam Date & Time: 22-Dec-2022 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH END SEMESTER EXAMINATIONS, DEC 2022

ENGINEERING MATHEMATICS III [MAT 2151]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions. Section Duration: 180 mins

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) Using finite difference method, solve for y, the differential equation

A) $y'' + 81y = 81x^2$ with the boundary values $y(0) = 0,\ \ y'(1) = 0$ and taking $h = \frac{1}{3}$. (4)

B) Solve $u_{xx} + u_{yy} - 2u_{xy} = 0$ using the transformations $v = x$, $z = x + y$. (3)

C) Find the acute angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (2, -1, 2). (3)

2) Verify Stoke's theorem for $\vec{A} = (2x - y)i - yz^2j - y^2zk$, where S is the upper half of the surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (4)

A)

B) Find the Half range cosine series of f(x) where $f(x) = \begin{cases} kx & 0 \le x \le \frac{l}{2} \\ k(l - x) & \frac{l}{2} \le x \le l \end{cases}$ (3)

C) Derive D'Alembert's solution of one dimensional wave equation. (3)

3)

A) Find the Fourier Transform of $e^{-a^2x^2}$; $a > 0$ and hence deduce $F\left(e^{\frac{-x^2}{2}}\right) = e^{\frac{-s^2}{2}}$ (4)

B) Solve $\frac{\partial^2 u}{\partial t^2} = \frac{1}{36}\frac{\partial^2 u}{\partial x^2}; 0 < x < 1; t > 0$ for 3-time levels with $k = 1$ subject to (3)

$$u(0,t) = u(1,t) = 0,\ u(x,0) = \frac{\partial u}{\partial t}(x,0) = \begin{cases} 12x, & 0 \le x \le \frac{1}{2} \\ 12(1-x), & \frac{1}{2} \le x \le 1 \end{cases}$$

C) Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 0 < x < 1,\ 0 < y < 1$ subject to the conditions $u(x,1) = 0$, $u(0,y) = 0,\ u(1,y) = 9(y - y^2),\ u(x,0) = 9(x - x^2)$. Choose $h = \frac{1}{3}$. (3)

4) A) Using divergence theorem, evaluate $\iint_S \vec{A}\,\overrightarrow{dS}$, where $\vec{A} = 4xi - 2y^2 j + z^2 k$ and S is the surface of the region bounded by $x^2 + y^2 = 4$, z = 0 an d z=3. (4)

B) Obtain the Fourier series expansion of $f(x) = x^2$ over the interval $(-\pi,\ \pi)$. Also deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\cdots$ (3)

C) Solve the PDE by separation of variables , $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$ (3)

5) A) Solve $32\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}\ ;\ 0 < x < 1;\ t > 0$ for 4 steps by explicit method with $h = \frac{1}{4};\ \lambda = \frac{1}{6}$ and $u(x,0) = 0 = u(0,t); u(1,t) = t$. (4)

B) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$ then prove that

i) $\nabla r^n = nr^{n-2}\ \vec{r}$

ii) $\nabla.(r^n\,\overrightarrow{r}) = (n+3)r^n$ (3)

C) Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data in $0 \le x \le 2\pi$. (3)

x^o	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

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