Exam Date & Time: 22-Dec-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

BTech III Semester-End Semester Examination-December 2022

ENGINEERING MATHEMATICS III [MAT 2156]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1)

A) Find the Fourier series expansion of function $f(x) = 3x^2$, where $-\pi < x < \pi$. Hence deduce that $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$. (3)

B) Compute up to second harmonics of the Fourier series of $y = f(x)$ given by the following table

x	0°	60°	120°	180°	240°	300°
y	0.8	0.6	0.4	0.7	0.9	1.1

(3)

C) Obtain the Fourier series for $f(x) = \begin{cases} l - x & 0 < x \le l \\ 0 & l \le x < 2l \end{cases}$. (4)

2) (3)

A)

Calculate the quartiles and quartile coefficient of skewness for the following data

Weight	70-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150
No. of persons	12	18	35	42	50	45	20	8

B) Calculate mean, median and mode for the following data:

Weight	0-10	10-20	20-30	30-40	40-50	50-60
Number of articles	14	17	22	26	23	18

(3)

C) Fit a second-degree parabola to the following data

x	1	2	3	4	5	6	7
y	-5	-2	5	16	31	50	73

(4)

3) Two lines of regression of x and y are $x = \frac{107}{20} + \frac{9}{20}y$ and $y = \frac{33}{5} + \frac{4}{5}x$.

A) Find the mean of x, mean of y and regression coefficient between x and y. (3)

B) Prove that the $\text{div}(r^n\vec{r}) = (n+3)r^n$ where $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$ and r denotes the magnitude of $\vec{r}$. (3)

C) (4)

Find $\text{div}(\vec{F})$ and $\text{curl}(\vec{F})$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.
Check whether $\vec{F}$ is solenoidal or not.

4) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$. (5)

A)

B) Using Green's theorem, evaluate

$$\int_C -yx^2dx + xy^2dy$$

where the curve C encloses the region bounded by x −axis and the circle $x^2 + y^2 = 25$ in the upper half plane. (2)

C) Solve the partial differential equation

$$x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} + u = 2$$ (3)

using indicated transformations $z = xy$ and $v = x$.

5) Solve the partial differential equation

A)
$$\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y,$$ (3)

where $z = 0$ when $y = \pi/2$ and $\frac{\partial z}{\partial y} = -2\sin y$ when $x = 0$.

B) (5)

Find d'Alembert's solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

where $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$.

C) Find the partial differential equation by eliminating arbitrary function from $z = (x+y)\phi(x^2 - y^2)$. (2)

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