



#### Instructions to Candidates:

- ❖ Answer **ALL** the questions.
- ❖ Missing data may be suitably assumed.
- ❖ Use of table of transforms is permitted.

**1A.** A discrete time signal  $x[n]$  is shown in Figure 1A.

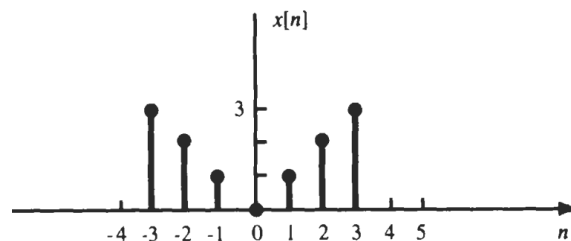


Figure 1A

Sketch and label the following:

- i)  $x[n]\delta[n-1]$
- ii)  $x[n/3]\{u[n+3] - u[n]\}$
- iii) Energy and power of  $x[n]$

(03)

**1B.** Determine if the system  $y[n] = x[n] \cos \omega n$  is linear, time-invariant, and stable. Justify your answer with necessary explanation.

(03)

**1C.** Evaluate the convolution sum  $y[n] = \beta^n u[n] * \alpha^n u[n-10]$ .

Given:  $|\beta| < 1$  and  $|\alpha| < 1$ .

(04)

**2A.** Find an Acausal signal whose Z-transform is given by

$$X[z] = \frac{2z^2 - 2.05z}{z^2 - 2.05z + 1}$$

(03)

**2B.** Use properties to find the DTFT of the following signal:  $x[n] = (\cos \frac{\pi}{4} n)(1/4)^n u[n-1]$

(03)

- 2C.** Consider an unknown causal discrete-time LTI system characterized by its impulse response  $h[n]$ . If an input  $x[n] = -\frac{1}{8}\left(\frac{1}{3}\right)^n u[n] - \frac{9}{8}3^n u[-n-1]$  is applied to the system, its response  $y[n]$  is available in terms of a pole-zero map in the complex Z-plane as shown in Figure 2C. Determine the impulse response of the system. Also, comment on the stability of the causal LTI system.

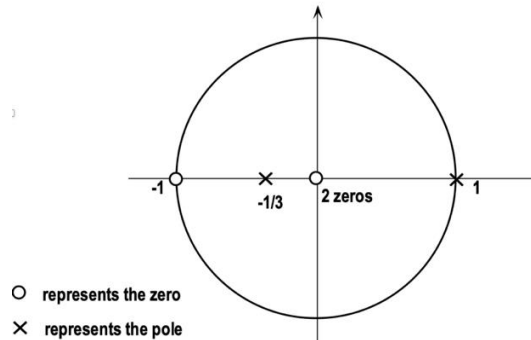


Figure 2C

(04)

- 3A.** The 8-point DFT of the sequence  $x[n] = [1, 1, 1, 1, 0, 0, 0, 0]$  is  $X[k] = [4, 1 - 2.41i, 0, 1 - 0.41i, 0, 1 + 0.41i, 0, 1 + 2.41i]$ . Using the properties of the DFT, compute the DFT of the sequence  $y[n] = [1, 0, 0, 0, 0, 1, 1, 1]$ .

(03)

- 3B.** Consider an analog signal represented as

$$x(t) = 25 \cos\left(60\pi t + \frac{\pi}{3}\right) + 15 \cos(250\pi t) \text{ applied to a sampling and reconstruction system.}$$

i) Suggest the minimum sampling rate that ensure  $y(t) = x(t)$ .

ii) Determine the value of B and the corresponding sampling rate  $F_s$  to get  $y(t) = B + 25 \cos\left(60\pi t + \frac{\pi}{3}\right)$ , where  $y(t)$  is the reconstructed signal.

(03)

- 3C.** Find the 8-point DFT of continuous time-signal  $x(t) = \sin(2\pi Ft)$  with  $F=100$  Hz using DIF-FFT algorithm. Draw the butterfly diagram and show all the values on the diagram. Assume sampling frequency of 400 Hz.

(04)

- 4A.** A digital IIR filter used for monitoring the signal amplitudes is represented as below:

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n] + \frac{1}{3}x[n-1].$$

Realize the given IIR filter using a filter structure that uses less memory elements.

(02)

- 4B.** It is required to make complete null in the frequency response corresponding to undesirable frequency of 60 Hz in recording application of heartbeat. The sampling frequency used is 300 Hz. Obtain the system function for a second order (i) FIR filter and (ii) pole-zero filter for the above purpose. In both the cases choose the normalizing factor such that  $|H(e^{-j\omega})| = 1$  for  $\omega = 0$ .

(05)

- 4C.** Write short notes on design of (i) FIR and (ii) IIR all pass filters using pole-zero approach. **(03)**
- 5A.** It is required to optimize signal-to-noise ratio and sensitivity of certain digital system performing low-pass filtering and satisfying the linear phase constraint. Achieve it by designing a suitable fourth-order filter with a cut-off frequency of  $\omega_c = \frac{\pi}{4}$ . Use Hanning window. Also, draw the linear phase structure for the designed filter. **(04)**
- 5B.** List the differences between impulse invariant and bilinear transformation techniques used in the design of IIR filters. **(02)**
- 5C.** For a certain biomedical application it is required to remove a band of frequencies. Design a Butterworth IIR filter using bilinear transformation technique to remove the band of frequencies described in the following specifications:
- $$0.8 \leq |H(e^{-j\omega})| \leq 1; \quad 0 \leq \omega \leq 0.2\pi$$
- $$|H(e^{-j\omega})| \leq 0.2; \quad 0.6\pi \leq \omega \leq \pi$$
- Consider sampling period  $T=1$ . **(04)**