Question Paper

Exam Date & Time: 31-Dec-2022 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SEVENTH SEMESTER B.TECH MAKEUP EXAMINATIONS, DEC 2022

Neural Networks and Fuzzy Logic [ICT 4052]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Derive the required relations for the recursive least-square estimation (RLS algorithm) of the weight vector in hybrid mode of learning in RBFN wherein the size of the hidden layer is a fraction of the training sample. (5)

B) Bayes classifier is a classical classifier which has conditional resemblance with the perceptron. As per the Bayes hypothesis testing procedure, we minimize the average risk, $\mathcal{R}$, which is defined as (3)

$$\mathcal{R} = c_{11}p_1 \int_{\mathcal{L}_1} p_{\mathrm{x}}(\mathrm{x}|\mathcal{C}_1)d\mathrm{x} + c_{22}p_2 \int_{\mathcal{L}_2} p_{\mathrm{x}}(\mathrm{x}|\mathcal{C}_2)d\mathrm{x} + \\ + c_{21}p_1 \int_{\mathcal{L}_2} p_{\mathrm{x}}(\mathrm{x}|\mathcal{C}_1)d\mathrm{x} + c_{12}p_2 \int_{\mathcal{L}_1} p_{\mathrm{x}}(\mathrm{x}|\mathcal{C}_2)d\mathrm{x}$$

where the various terms are defined as follows:

p_i: a priori probability that the observation vector x is drawn from subspace $\mathcal{L}_i$ and $p_1 + p_2 = 1$

c_{ij}: cost of deciding in favor of class $\mathcal{C}_i$ represented by subspace $\mathcal{L}_i$ when class $\mathcal{C}_j$ is true

$p_{\mathrm{x}}(\mathrm{x}|\mathcal{C}_i)$: conditional probability density function of the random vector X, given that the observation vector x is drawn from subspace $\mathcal{L}_i$

Derive the required conditions for the Bayes classifier.

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C) (2)

Compute the weight estimate for the given data using *least-square filter*

$$\mathrm{x}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, d_1 = 1; \mathrm{x}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, d_2 = 2; \mathrm{x}_3 = \begin{pmatrix} 5 \\ 6 \end{pmatrix}, d_3 = 3.$$

Assume that the Gauss-Newton method has been used as the optimization method.

2) A) For the data set given in Table Q.2A, design a polynomial learning machines whose inner product kernel is given by (5)

$$K(\mathrm{x}, \mathrm{x}_i) = (\mathrm{x}^T\mathrm{x}_i + 1)^2$$

and compute the optimum margin of separation, ρ for this machine.

Table: Q.2A

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Input Vector, X	Desired Response, d
$(-1,-1)$	-1
$(-1,+1)$	$+1$
$(+1,-1)$	$+1$
$(+1,+1)$	-1

B) Briefly explain the following heuristics for making the back-propagation algorithm perform better: (3)

i) Stochastic versus batch mode
ii) Maximizing information content
iii) Activation function
iv) Normalizing the inputs
v) Initialization
vi) Learning rates

C) The method of steepest descent converges to the optimum solution $\mathbf{w}^*$ slowly. Furthermore, the learning-rate parameter η has a profound influence on its convergence. State the effect of learning-rate parameter on the behavior of steepest-descent method, and name a method which overcomes its limitations. (2)

3) (5)

A)

Use the back-propagation algorithm for computing a set of synaptic weights and bias levels for a neural network as shown in Fig.Q.3A to solve the XOR problem. Assume the use of a logistic function for the nonlinearity.

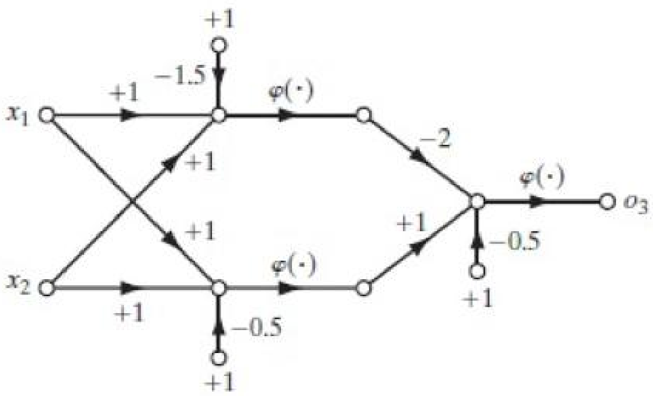

Fig.: Q.3A

B) (3)

We want to set up a database of documents that cover a certain broad scientific subject to various degrees of depth. We are interested in the relationship, for each individual document, between a researcher's amount of expertise in this subject and relevance of that document to the teacher's scientific endeavors. A "relevant" document is defined as

$$\text{"Relevant"} = V = \left\{ \frac{0.0}{irrelevant} + \frac{0.2}{tangential} + \frac{1.0}{relevant} + \frac{1.0}{crucial} \right\}.$$

A "knowledgeable" researcher is defined as

$$\text{"Knowledgeable"} = K = \left\{ \frac{0.0}{novice} + \frac{0.3}{student} + \frac{0.8}{graduate} + \frac{1.0}{expert} \right\}.$$

Consider a certain very in-depth document. A knowledgeable researcher will find this document relevant to the research at hand. There is a relationship between K and V.

i) Find the relation IF K, THEN V for this in-depth document using classical implication.

ii) A "freshman" researcher would have different view of the same document. A freshman could be described by another fuzzy set on the universe of researchers as

$$\text{"freshman"} = F = \left\{ \frac{0.5}{novice} + \frac{0.7}{student} + \frac{0.0}{graduate} + \frac{0.0}{expert} \right\}.$$

Find the relevancy of the in-depth document to this freshman using max-min composition on the relation found in (i).

C) Consider a fuzzy set, $A = \left\{\frac{0.2}{2} + \frac{0.4}{3} + \frac{0.8}{4} + \frac{1}{5} + \frac{0.9}{6} + \frac{0.5}{7} + \frac{0.1}{8}\right\}$. Find the following (2)

i) Core element of the set

ii) Boundary elements of the set

iii) Support element of the set

iv) Whether the given fuzzy set a normal fuzzy set?

4) (5)

A)

Two companies bid for a contract. A committee has to review the estimates of those companies and give reports to its chairperson. The reviewed reports are evaluated on a non-dimensional scale and assigned a weighted score that is represented by a fuzzy membership function, as illustrated by the two fuzzy sets A_1 and A_2, in Fig. Q.4A. The chairperson is interested in the lowest bid, as well as a metric to measure the combined "best" score. For the logical union of the membership function shown we want to find the defuzzified quantity. Use centroid method to calculate the defuzzified value, z^*.

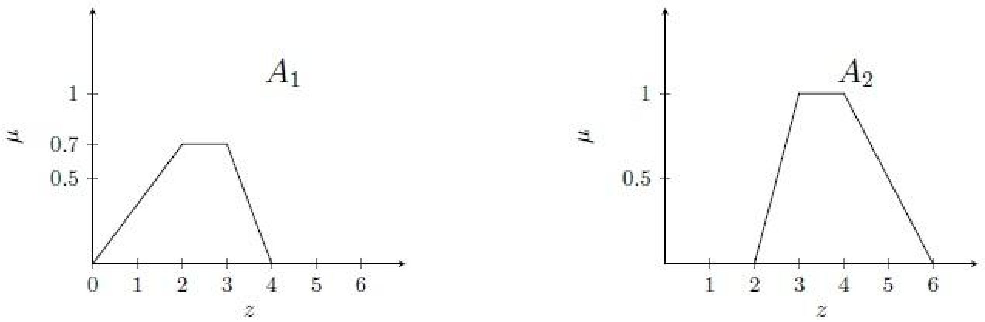

Figure: Q.4A

B) Two fuzzy sets A and B, both defined on X, are given in Table Q.4B. Show that the (3)

Table: Q.4B

$\mu(x_i)$	x_1	x_2	x_3	x_4	x_5	x_6
$\tilde{A}$	0.1	0.6	0.8	0.9	0.7	0.1
$\tilde{B}$	0.9	0.7	0.5	0.2	0.1	0

λ-cut set obey the following properties:

i) $(\tilde{A} \cup \tilde{B})_{0.3} = A_{0.3} \cup B_{0.3}$

ii) $(\tilde{A} \cap \tilde{B})_{0.4} = A_{0.4} \cap B_{0.4}$

iii) $(\overline{\tilde{A}})_{0.6} \neq \overline{A}_{0.6}$.

C) (2)

Consider two fuzzy sets A and B defined as

$$A = \left\{ \frac{1}{1.0} + \frac{0.75}{1.5} + \frac{0.3}{2.0} + \frac{0.15}{2.5} + \frac{0}{3.0} \right\} \quad B = \left\{ \frac{1}{1.0} + \frac{0.6}{1.5} + \frac{0.2}{2.0} + \frac{0.1}{2.5} + \frac{0}{3.0} \right\}$$

For these two fuzzy sets, find the following:

(i) $A|B$ (ii) $\overline{A \cap B}$.

5) A) For the data shown in Table Q.5A, show the first iteration of back propagation algorithm (5) in trying to compute the membership values for the input variables x_1, x_2 and x_3 in the output region R^1 and R^2. Use a $3 \times 3 \times 2$ neural network. Assume a random set of weights

for your neural network.

Table: Q.5A

x_1	x_2	x_3	R^1	R^2
2.5	1.0	0.8	1.0	0.0

B) With respect to imaging radars when we speak of "resolution," we are referring to the "fitness" of our ability to distinguish closely spaced targets. We may have "high" resolution radars or "low" resolution radars, or some other fuzzy description of their resolution ability. Define the following fuzzy sets on the universe of radar image resolution as measures in meters: (3)

$$\text{“High resolution} = \left\{\frac{1}{0.1} + \frac{0.9}{0.3} + \frac{0.5}{1} + \frac{0.2}{3} + \frac{0.1}{10} + \frac{0}{30}\right\}$$

$$\text{“Low resolution} = \left\{\frac{0}{0.1} + \frac{0.1}{0.3} + \frac{0.3}{1} + \frac{0.7}{3} + \frac{0.9}{10} + \frac{1}{30}\right\}.$$

Find membership function for the following linguistic phrases:

i) Not high resolution and not low resolution

ii) Low resolution or not very high resolution

iii) High resolution and not very, very high resolution.

C) (2)

Prove the given statement by contradiction

$$((p \to \neg q) \wedge (q \vee \neg r) \wedge (r \wedge \neg s)) \to \neg p$$

where $\neg$ indicates complement (negation).

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