Question Paper

Exam Date & Time: 23-Nov-2022 (09:00 AM - 12:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SEVENTH SEMESTER B.TECH END SEMESTER EXAMINATIONS, NOV 2020

Neural Networks and Fuzzy Logic [ICT 4052]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) The perceptron bears a certain relationship to a classical pattern classifier known as the Bayes classifier. Show that a Bayes classifier reduces to a linear classifier, when the environment is Gaussian. The multivariate Gaussian probability density function is defined as (5)

$$f(\mathbf{x}) = \frac{1}{(2\pi)^{m/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)$$

where m is the dimensionality of the observation vector $\mathbf{x}$, $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the parameters of the Gaussian function.

B) Designing *Radial Basis Function* networks as per the interpolation theory has implementation difficulties. With a neat diagram explain how these limitations can be overcome. (3)

C) Derive the following results for weight updation as per the *least-mean-square* (LMS) algorithm, $\hat{\mathbf{w}}(n+1) = \hat{\mathbf{w}}(n) + \eta\mathbf{x}(n)e(n)$. (2)

2) A) (5)

Consider the case of a hyperplane for linearly separable patterns, which is defined by the equation

$$\mathbf{w}^T\mathbf{x} + b = 0$$

where $\mathbf{w}$ denotes the weight vector, b denotes the bias, and $\mathbf{x}$ denotes the input vector. The hyperplane is said to correspond to a *canonical pair* $(\mathbf{w}, b)$ if, for the set of input patterns $\{\mathbf{x}_i\}_{i=1}^N$, the additional requirement

$$\min_{i=1,\ldots,N} |\mathbf{w}^T\mathbf{x}_i + b| = 1$$

is satisfied. Show that this requirement leads to a margin of separation between the two classes equal to $2/||\mathbf{w}||$.

B) As per the interpolation theory, in a RBF network when $m_1 = N$, the weight is obtained by $\mathbf{W} = \mathbf{\Phi}^{-1}\mathbf{D}$. Show that when $m_1 < N$, the relation for weight is given by (3)

$$\mathbf{W} = (\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\mathbf{\Phi}^T\mathbf{D}$$

where m_1 is the dimensionality of hidden space, N is the number of training examples, $\mathbf{\Phi}$ is the interpolation matrix, and $\mathbf{D}$ is the desired vector.

C) What do you understand by the term *kernel trick*? (2)

3) A) Consider a fully connected feedforward 4-3-2-1 network. Construct an architectural graph for this network. Apply back-propagation algorithm to this network, and write the relation for each synaptic weight after one iteration of the back-propagation algorithm. Assume that each neuron in the network uses logistic function as an activation function. (5)

B) (3)

An engineer is testing the properties, strength and weight of steel. Suppose he has two fuzzy sets A, defined on a universe of three discrete strengths, $\{s_1, s_2, s_3\}$, and B, defined on a universe of three discrete weights, $\{w_1, w_2, w_3\}$. Suppose A and B represent a "high-strength steel" and a "near-optimum weight," respectively, as given below

$$A = \left\{\frac{1}{s_1} + \frac{0.5}{s_2} + \frac{0.2}{s_3}\right\}, \ B = \left\{\frac{1}{w_1} + \frac{0.5}{w_2} + \frac{0.3}{w_3}\right\}.$$

i) Find the fuzzy relation for the Cartesian product of A and B (i.e., $R = A \times B$). Here the Cartesian product represent the strength-to-weight characteristics of a near maximum steel quality.

ii) Suppose we introduce another fuzzy set, C, which represents a set of "moderately good" steel strengths, say, the following:

$$C = \left\{\frac{0.1}{s_1} + \frac{0.6}{s_2} + \frac{1}{s_3}\right\}.$$

Find $C \circ R$

C) Consider the fuzzy sets $A, B : \{1, 2, \ldots, 10\} \to [0, 1]$ defined as (2)

$$A(x) = \left\{\frac{0.2}{2} + \frac{0.7}{3} + \frac{1}{4} + \frac{0.7}{5} + \frac{0.2}{6}\right\}, \text{ and } B(x) = \left\{\frac{0.3}{4} + \frac{0.5}{5} + \frac{0.8}{6} + \frac{1}{7} + \frac{0.5}{8} + \frac{0.2}{9}\right\}.$$

Calculate $A|B$ and $\overline{A \wedge B}$.

4) (5)

A)

A railroad company intends to lay a new rail line on Konkan route. The whole area through which the new line is passing must be purchased for right-of-way consideration. It is surveyed in three stretches, and the data are collected for analysis. The surveyed data for the road are given by the sets, A_1, A_2, and A_3, where the sets are defined on the universe of right-of-way widths, in meters. For the railroad to purchase the land, it must have an assessment of the amount of land to be bought. The three surveys on right-of-way width are ambiguous, however, because some of the land along the proposed railway route is already in public domain and will not need to be purchased. Additionally, the original surveys are so old that some ambiguity exists on boundaries and public right-of-way for old utility lines and old roads. The three fuzzy sets, A_1, A_2, and A_3 are shown in Figure Q.4A, represent the uncertainty in each survey as to the membership of right-of-way width, in meters, in privately owned land. Calculate the single most nearly representative right-of-way width using centroid method.

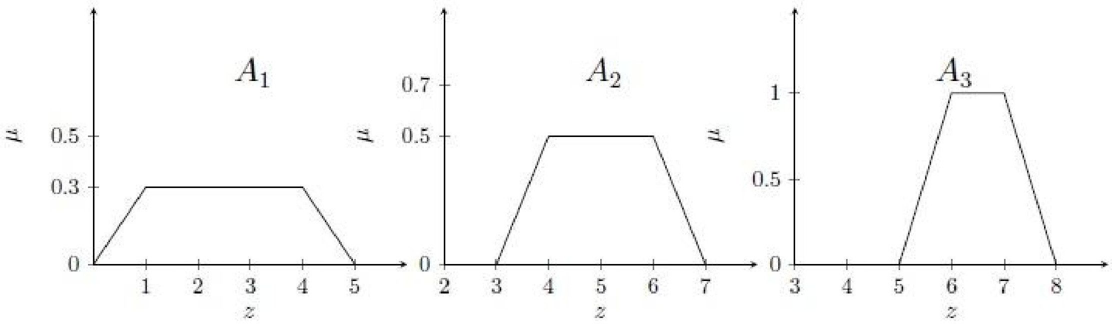

Figure: Q.4A

B) Two fuzzy sets A and B both defined on X, are as follows: Express the following λ-cut (3)

$\mu(x_i)$	x_1	x_2	x_3	x_4	x_5	x_6
A	0.1	0.6	0.8	0.9	0.7	0.1
B	0.9	0.7	0.5	0.2	0.1	0

sets using Zadeh's notation:

(i) $(\overline{A})_{0.6}$ (iii) $(A \cup \overline{A})_{0.6}$ (v) $(\overline{A \cap B})_{0.6}$

(ii) $(B)_{0.4}$ (iv) $(B \cap \overline{B})_{0.5}$ (vi) $(\overline{A} \cup \overline{B})_{0.7}$

C) For the inference rule, $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ prove that the rule is a tautology. (2)

5) For the data shown in the accompanying table, show the first iteration in trying to (5)
A) compute the membership values for the input variables x_1, x_2, x_3 and x_4 in the output region R^1, R^2 and R^3. Use a $4 \times 3 \times 3$ neural networks with a random set of weights.

x_1	x_2	x_3	x_4	R^1	R^2	R^3
10	0	-4	2	0	1	0

B) In risk assessment, we deal with characterizing uncertainty in assessing the hazard to human health posed by various toxic chemicals. Because the phramocokinetics of the human body are very difficult to explain for long-term chemical hazards, such as chronic exposure to lead or to cigarette smoke, hazards can sometimes be uncertain because of scarce data or uncertainty in the exposure patterns. Let us characterize hazard linguistically with two terms: "low" hazard and "high" hazard: (3)

$$\text{"Low" hazard} = \left\{ \frac{1}{1} + \frac{0.8}{2} + \frac{0.5}{3} + \frac{0.1}{4} + \frac{0}{5} \right\}$$

$$\text{"High" hazard} = \left\{ \frac{0}{1} + \frac{0.2}{2} + \frac{0.4}{3} + \frac{0.9}{4} + \frac{1}{5} \right\}.$$

Find the membership functions for the following linguistic expressions:

i) low hazard and not high hazard
ii) very high hazard and not low hazard
iii) low hazard or high hazard.

C) Using the inference approach, find the membership values for each of the triangular shapes (I,R,IR,E,T) for each of the following triangles: (2)

(i) $80^0, 70^0, 30^0$
(ii) $65^0, 60^0, 55^0$.

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