Question Paper

Exam Date & Time: 31-Dec-2022 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SEVENTH SEMESTER B.TECH (INFORMATION TECHNOLOGY/COMPUTER & COMMUNICATION ENGINEERING) DEGREE MAKEUP EXAMINATIONS, DEC 2022

Pattern Recognition [ICT 4053]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Fraudulent transactions are more likely when the cardholder travels abroad since tourists are prime targets for thieves. More precisely, 1% of transactions are fraudulent when the cardholder travels, whereas only 0.4% of the transactions are fraudulent when she is not traveling. On average, 5% of all transactions happen while the cardholder travels. If a transaction is fraudulent, then the likelihood of a foreign purchase increases unless the cardholder happens to be traveling. More precisely, when the cardholder is not traveling, 10% of the fraudulent transactions are foreign purchases, whereas only 1% of the legitimate transactions are foreign purchases. On the other hand, when the cardholder travels, 90% of the transactions are foreign purchases, regardless of the legitimacy of the transactions. (5)

a) Construct a Bayes Network to identify fraudulent transactions. Show the graph defining the network and the Conditional Probability Tables associated with each node in the graph. This network should encode the information stated above. The network should contain the following nodes corresponding to the following binary random variables:

- **Fraud** – current transaction is fraudulent
- **Trav** – the cardholder, is currently traveling
- **FP** – current transaction is a foreign purchase.

b) Find $P(\text{fraud=false}|\text{foreign-purchase=true})$.

B) (3)

Given $\lambda_{11} = \lambda_{22} = 0, \lambda_{12} = \lambda_{21} = 1, p(\omega_1|\text{X}) = 0.196$ and $p(\omega_2|\text{X}) = 0.804$, compute $R(a_1|\text{X})$, and $R(a_2|\text{X})$. To which class would a minimum risk classifier classify an unknown sample X?

C) Discuss the nature of the decision boundary when all the classes have the same covariance matrix with arbitrary values. (2)

2) A) Using Parzen windows (pictorial representation), find $p_n\binom{3}{5}$ for 2D space on the following data (5)

$$D = \left\{ \binom{2}{2}, \binom{2}{5}, \binom{3}{3}, \binom{4}{2}, \binom{4}{4}, \binom{4}{4}, \binom{4}{5}, \binom{5}{6}, \binom{6}{1}, \binom{6}{3}, \binom{6}{5}, \binom{7}{2}, \binom{7}{4} \right\}.$$

B) Consider a model with a parameter vector $\boldsymbol{\theta} = (\boldsymbol{\mu}, \boldsymbol{\Sigma})$, having a Gaussian distribution over its data. How will the maximum likelihood method estimate the unknown $\boldsymbol{\mu}$, given the $\boldsymbol{\Sigma}$? (3)

C) How are minimum squared error criteria relevant in fetching a weight vector in the case of linearly non-separable classes? (2)

C) How are minimum squared error criteria relevant in fetching a weight vector in the case of linearly non-separable classes? (2)

3) Consider following data elements for two class problem (5)

A)

$$\left\{ \begin{pmatrix} 7.1 \\ 4.2 \end{pmatrix}, \begin{pmatrix} -1.4 \\ -4.3 \end{pmatrix}, \begin{pmatrix} 4.5 \\ 0 \end{pmatrix}, \begin{pmatrix} 6.3 \\ 1.6 \end{pmatrix}, \begin{pmatrix} 4.2 \\ 1.9 \end{pmatrix}, \begin{pmatrix} 1.4 \\ -3.2 \end{pmatrix}, \begin{pmatrix} 2.4 \\ -4 \end{pmatrix}, \begin{pmatrix} 2.5 \\ -6.1 \end{pmatrix}, \begin{pmatrix} 8.4 \\ 3.7 \end{pmatrix}, \begin{pmatrix} 4.1 \\ -2.2 \end{pmatrix} \right\}$$

and

$$\left\{ \begin{pmatrix} -3 \\ -2.9 \end{pmatrix}, \begin{pmatrix} 0.5 \\ 8.7 \end{pmatrix}, \begin{pmatrix} 2.9 \\ 2.1 \end{pmatrix}, \begin{pmatrix} -0.1 \\ 5.2 \end{pmatrix}, \begin{pmatrix} -4.2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1.3 \\ 3.7 \end{pmatrix}, \begin{pmatrix} -3.4 \\ 6.2 \end{pmatrix}, \begin{pmatrix} -4.1 \\ 3.4 \end{pmatrix}, \begin{pmatrix} -5.1 \\ 1.6 \end{pmatrix}, \begin{pmatrix} 1.9 \\ 5.1 \end{pmatrix} \right\}$$

belonging to class 1 and 2 respectively. If the initial vector is $\binom{0.9}{1}$ with $w_0 = -5$ and $\eta = 0.3$, using perceptron criteria, calculate the weight vector at the end of the second iteration.

B) Given a criterion defined as $J_S(a, b) = ||Ya - b||^2$, how does the Ho-Kashyap rule minimize J_S? (3)

C) How should one measure the similarity between samples in a cluster? (2)

4) (5)

A)

For the data given in Table.Q.4A demonstrate the formation of two clusters using a single linkage algorithm.

Table:Q.4A

Sample	x_1	x_2
1	7.1	4.2
2	-1.4	-4.3
3	4.5	0
4	6.3	1.6
5	4.2	1.9
6	1.4	-3.2

B) (3)

Let $x_1 = \binom{4}{5}, x_2 = \binom{1}{4}, x_3 = \binom{0}{1}$ and $x_4 = \binom{5}{0}$, and consider the following partitions:

$$\mathcal{D}_1 = \{x_1, x_2\}, \mathcal{D}_2 = \{x_3, x_4\}, \mathcal{D}_1 = \{x_1, x_2, x_3\}.$$

Compute the following

1. Within cluster scatter matrix, S_W
2. Between cluster scatter matrix, S_B, and
3. Total scatter matrix, S_T.

C) Suggest a solution when an exhaustive search for clustering becomes infeasible. (2)

5) (5)

A)

Consider a HMM, $\boldsymbol{\theta} = (A, B)$, where A, and B are the transition and emission probability matrices which are given by

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.8 & 0.1 & 0.0 & 0.1 \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.1 & 0.2 \\ 0 & 0.1 & 0.1 & 0.7 & 0.1 \\ 0 & 0.5 & 0.2 & 0.1 & 0.2 \end{pmatrix}.$$

Assume that the initial state at $t = 0$ be ω_1. What is the probability that the model generates the sequence $\mathbf{V}^7 = \{v_3, v_1, v_3, v_2, v_2, v_4, v_0\}$? Assume that the matrix indexes begin at 0. [Note: v_0 denotes the visible symbol emitted by the accepting/final state]

B) List and describe three central issues in HMMs. Provide generic example scenario for each of the issues. (3)

C) Consider the given algorithm, and write the HMM backward algorithm. (2)

HMM Forward

1: **initialize** $\omega(1), t = 0, a_{ij}, b_{jk}, \mathbf{V}^T, \alpha(0) = 1$
2: **for** $t = t + 1, \ldots, T$ **do**
3: $\alpha_j(t) \leftarrow b_{jk}v(t) \sum_{i=1}^{c} \alpha_i(t-1)a_{ij}$
4: **return** $P(\mathbf{V}^T) \leftarrow \alpha_0(T)$

-----End-----