MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

VII SEMESTER B.TECH

END SEMESTER EXAMINATIONS, NOVEMBER 2022

SUBJECT: GRAPHS AND MATRICES [MAT 4054] (PE)

Instructions to Candidates:

- Answer ALL the questions.

Date: **28-11-2022** Time: **02.00 PM – 05.00 PM** Max. Marks: **50**

1A. Show that a graph is bipartite if and only if it contains no odd cycles **(3 Marks)**

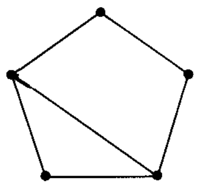
1B. Let G be a connected cubic graph. Show that G has a cut vertex if and only if G has a bridge **(3 Marks)**

1C. Let G be a connected graph. If $diam(\bar{G}) \geq 3$, then show that $diam(\bar{G}) \leq 3$. Hence deduce that the diameter of a self-complementary graph is either 2 or 3 **(4 Marks)**

2A. Let G be a simple (p, q) graph. If G is a tree, then show that p = q + 1. Hence deduce that every non-trivial tree has at least two pendant vertices **(3 Marks)**

2B. Show that the complete graph K_5 on 5 vertices and the complete bipartite graph $K_{3,3}$ are not planar graphs **(3 Marks)**

2C. Obtain chromatic polynomial of the following graph



Use the recurrence relation f(G, t) = f(G + e, t) + f(G.e, t) **(4 Marks)**

3A. Let G be a Hamiltonian graph and S be non-empty subset of vertex set of G. Then show that c(G-S) ≤ n(S) where c(G-S) represents number of components in G – S and n(S) denotes number of vertices in S. Give an example to show that this condition is not sufficient to say that a graph is Hamiltonian **(3 Marks)**

3B. For a non-trivial connected graph G with p points show that $\alpha_0 + \beta_0 = p$, where α_0 and β_0 denote point covering number and point independence number respectively **(3 Marks)**

3C. Let G be a tree with vertex set {1, 2, 3, …, n}. Let Q be the {0, 1, -1} incidence matrix of G and let Q_n be the reduced incidence matrix obtained by deleting row n of Q. Show that $Q_n^{-1} = P_n$. where P_n is the path matrix **(4 Marks)**

4A. Let G be a graph with {0, 1, -1} incidence matrix Q. Show that Q is totally unimodular **(3 Marks)**

4B. For any graph G, show that $\chi(G) \leq 1 + \lambda_1(G)$, where $\chi(G)$ denotes the chromatic number and $\lambda_1(G)$ denotes the largest eigen value of the adjacency matrix of G **(3 Marks)**

4C. Let G be a graph with n vertices, m edges and let λ_1 be the largest eigen value of G. Then show that $\lambda_1 \leq \sqrt{\frac{2m(n-1)}{n}}$ **(4 Marks)**

5A. For any positive integers m, n show that the eigen values of $K_{m,n}$ are $\sqrt{mn}$, $-\sqrt{mn}$ and 0 with respective multiplicities 1, 1 and m + n -2 **(3 Marks)**

5B. Let L denote the Laplacian matrix of a simple graph G. For any vector x, show that

$$x^T L x = \sum_{i\sim j}\left(x_i - x_j\right)^2$$ **(3 Marks)**

5C. Let G be a simple graph with at least one edge. Let λ_1 be the largest eigen value of the Lapacaian matrix of G. Show that $\lambda_1 \geq \Delta(G) + 1$, where λ_1 denotes the largest eigen value of the Laplacian matrix and $\Delta(G)$ denotes the maximum degree of G **(4 Marks)**

********** GOOD LUCK **********