Question Paper

Exam Date & Time: 31-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - DECEMBER 2022
I SEMESTER B.Sc. (Applied Sciences) in Engg.
MATHEMATICS-I [IMA 111 - S2]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1) (4)

A) If $y^{1/m} + y^{-1/m} = 2x$ show that
$(x^2-1)y_{n+2} + x(2n+1)y_{n+1} + (n^2-m^2)y_n = 0$.

B) (3)

Find the angle of intersection of the curves : $r^2 sin2\theta = 4$, $r^2 = 16sin2\theta$

C) (3)

A curve is given by $x = asin\theta, y = bcos2\theta$. Find the radius of curvature at $\theta = \frac{\pi}{3}$

2) (4)

A) If $r^2 = x^2 + y^2 + z^2, v = r^m$ prove that
$v_{xx} + v_{yy} + v_{zz} = m(m+1)r^{m-2}$

B) (3)

Find the circle of curvature for the curve $y^2 = 4ax$ at $(at^2, 2at)$.

C) (3)

Expand $f(x,y) = e^x siny$ in powers of x and y using Taylor's series up to third degree terms.

3) (4)

A) Find the extreme values of the function
$f(x,y) = x^3 + y^3 - 3x - 12y + 2.$

B) (3)

Evaluate using reduction formula: $\int_0^1 x^2(1-x^2)^{\frac{3}{2}}\,dx$

C) (3)

Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)(n+2)}$

4) (4)

A) Find the area bounded by the curve $xy^2 = 4a^2(2a - x)$,a > 0 and its asymptote

B) (3)

Evaluate using reduction formula: $\int_0^1 \frac{x^7 dx}{\sqrt{1-x^4}}$

C) (3)

Evaluate $\lim_{x \to \frac{\pi}{2}} (sinx)^{tanx}$

5) (4)

A) (a) State and prove Rolle's theorem.
(b) Verify Rolle's theorem for the function
$f(x) = e^x\ (sinx - cosx)$ in $[\frac{\pi}{4}, \frac{5\pi}{4}]$.

B) (3)

If the limit of $\frac{sin2x + asinx}{x^3}$ as x tends to zero, be finite, find the value of 'a' and the limit

C) (3)

Expand $tan^{-1}x$ in powers of $(x - 1)$ up to terms containing x^4.

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