Question Paper

Exam Date & Time: 20-Dec-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - DECEMBER 2022
I SEMESTER B.Sc. (Applied Sciences) in Engg.

MATHEMATICS - 1 [IMA 111]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1)

A) If $y = \cos(m \sin^{-1} x)$ show that $(1-x^2)y_{n+2} - x(2n+1)y_{n+1} + (m^2 - n^2)y_n = 0$. Find $y_n(0)$ (4)

B) Find the angle of intersection of the curves $r = \frac{a}{1+\theta^2}$, $r = \frac{a\theta}{1+\theta}$ (3)

C) If ρ be the radius of curvature at any point P on the parabola $y^2 = 4ax$ and S be the focus then show that the radius of curvature at P varies as $(SP)^{3/2}$. (3)

2)

A) If $u = f(r)$ and $x = r\cos\theta, y = r\sin\theta$ prove that $u_{xx} + u_{yy} = f''(r) + \frac{f'(r)}{r}$ (4)

B) Find the circle of curvature for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $(\frac{a}{4}, \frac{a}{4})$. (3)

C) Expand $f(x,y) = \sin(xy)$ in powers of $(x-1)$ and $(y - \frac{\pi}{2})$ using Taylor's series up to second degree terms. (3)

3)

A) Find the extreme values of the function $f(x,y) = xy(a - x - y)$. (a > 0) (4)

B) Test the convergence of the series by comparison test $\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n^2\sqrt{n}}$ (3)

C) Evaluate using reduction formula: $\int_0^a \frac{x^4 dx}{\sqrt{a^2 - x^2}}$ (3)

4) (4)

A) Trace the curve: $xy^2 = 4a^2(2a - x)$,a > 0

B) Evaluate using reduction formula: $\int_0^2 x^3 \sqrt{2x - x^2}\, dx$ (3)

C) Evaluate $\lim_{x \to 0} \left(\frac{a^x + b^x}{2}\right)^{\frac{1}{x}}$ (3)

5) (4)

A) (a) State and prove Cauchy's mean vale theorem.

(b) Verify Cauchy's mean value theorem for the function $f(x) = logx$ and $g(x) = \frac{1}{x}$ in [1,e]

B) Evaluate $\lim_{x \to 0} \frac{sinx - x + \frac{x^3}{6}}{x^5}$ (3)

C) Obtain the Maclaurin's series expansion of $\log(1 + e^x)$ up to terms containing x^4. (3)

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