Question Paper

Exam Date & Time: 31-Jan-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - DECEMBER 2022
III SEMESTER B.Sc (Applied Sciences) in Engg.

Mathematics -III [IMA 231 - S2]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1) (4)

A) Using the method of variation of parameters, solve $(D^2 + 4)y = tan\, 2x$.

B) (3)

Solve $x^2y'' - 2xy' - 4y = x^4$ for $y(x)$.

C) (3)

Solve

$$\frac{dx}{dt} = 3x - 4y,\ \frac{dy}{dt} = x - y, \quad \text{for } x(t) \text{ and } y(t).$$

2) (3)

A) Calculate $L(e^{-t}\, cos^2\, t)$

B) (3)

Find the Laplace transform of $f(t) = \frac{e^{-at} - e^{-bt}}{t}$.

C) (4)

Calculate the following

$L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$, here L^{-1} denotes the inverse Laplace transform.

3) (4)

A) Solve the following initial value problem (IVP) using Laplace transform

$$y'' - 10y' + 9y = 5t, with$$

$$y(0) = -1, y'(0) = 2.$$

B) (3)

Solve the following initial value problem (IVP)

Solve the following initial value problem (IVP)

$$y'' + y' - 6y = 0, with$$

$$y(0) = 1, y'(0) = 0.$$

C) (3)

Evaluate $\int_0^{\infty} \frac{\sin t}{t}\, dt.$

4) (3)

Show that $f(z) = \sin \bar{z}$ is analytic and find $f'(z)$.

A)

B) (4)

Show that $v(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find its harmonic conjugate and corresponding analytic function $f(z)$.

C) (3)

Evaluate $\int_c (\bar{z})^2 dz$,

(i) along a line from $z = 0$ to $z = 2 + i$,

(ii) along the real axis to 2 and then vertically to $2+i$.

5) (4)

Find $y(0.2)$ for $y' = \frac{x-y}{2}, y(0) = 1$, with step size 0.01 using

A) Runge-Kutta method of order four.

B) (3)

Find Taylor series expansion of $f(z) = \sin z$ about $z = 0$, using the definition. Also, find series expansion for $\sin z^2$.

C) (3)

Expand $f(z) = \frac{1}{z^2 - 2z - 3}$ about the point $z = 0$.

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