Question Paper

Exam Date & Time: 19-Dec-2022 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - DECEMBER 2022
III SEMESTER B.Sc (Applied Sciences) in Engg.

Mathematics - III [IMA 231]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1) (4)

A) Using the method of variation of parameters, solve

$$(D^2 - 9)y = \frac{2}{1+e^x}.$$

B) (3)

Solve $(3x+2)^2y'' - (3x+2)y' + y = 3x + 4.$

C) (3)

Solve

$$\frac{dx}{dt} = 3x - 4y,\ \frac{dy}{dt} = x - y,$$

for $x(t)$ and $y(t)$.

2) (3)

Find Laplace transform

A) $L(e^{3t} - 2e^{-2t} + sin2t + cos3t + sinh3t - 3cosh5t + 6)$

B) (3)

Find Laplace transform of

$\int_0^t \int_0^t \frac{\sin u}{u} du.$

C) (4)

Solve the following initial value problem (IVP) using Laplace transform

$$y'' - 10y' + 9y = 5t, with$$

$$y(0) = -1, y'(0) = 2.$$

3) (4)

A) Show that $v(x,y) = -\sin x \sin h\, y$ is a harmonic function.
Find the harmonic conjugate of $v(x,y)$ and the corresponding analytic function f(z).

B) (2)

B) (3)

Show that the function $f(z) = z^2$ is analytic and find $f'(z)$.

C) (3)

Evaluate $\int_c z^2 dz$,

(i) along a line from $z = 0$ to $z = 3 + i$,

(ii) along a line from $z = 0$ to $z = 3$ *and* then vertically $z = 3$ to $z = 3 + i$.

4) (4)

A) Use the Runge-Kutta method with $h = 0.1$ to solve the initial value problem

$$y' + 2y = x^3 e^{-2x}, y(0) = 1,$$

at $x = 0.1, 0.2$.

B) (3)

Evaluate $\int_c \frac{z}{(z^2+1)(z^2+9)} dz$, where

(i) $c: |z| = 2$,

(ii) $c: |z - 2| = 2$.

C) (3)

Expand $f(z) = \frac{z}{(z+3)(z+1)}$ in the region $|z - 1| < 2$.

5) (3)

A) Find Taylor series expansion of $f(z) = ze^z$ about $z = 1$, using definition.

B) (4)

Calculate the following

$L^{-1}\left(\frac{s^3+6s^2+14s}{(s+1)^4}\right)$, here L^{-1} denotes the inverse Laplace transform.

C) (3)

Solve $y'' + 4y = x^2$ for $y(x)$.

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