Reg. No.										

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

I SEMESTER M.TECH. (ENVIORNMENTAL ENGG.)

END SEMESTER EXAMINATIONS Jan 2023

SUBJECT: COMPUTATIONAL METHODS AND OPTIMIZATION TECHNIQUES [MAT 5160]

Time: 3 hrs Max. Marks: **50**

Instructions to Candidates:

- Answer ALL the questions. Use of STATISTICAL TABLES is permitted

1A.	Maximize z = x + 1.5 y given $x \geq 0$ $y \geq 0$ subjects to the conditions $x + 2y \leq 160$, $3x + 2y \leq 240$ by graphical method.	**3M**
1B.	Find the pdf of $Y = 8X^3$ if X has the pdf $f(x) = \begin{cases} 2x; 0 \leq x \leq 1 \\ 0; elsewhere \end{cases}$.	**3M**
1C.	The function u satisfies the equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$ and the conditions , $u(x, 0) = 100(x - x^2)$, and $u_t(x,0) = 0, 0 \leq x \leq 1, u(0,t) = u(1,t) = 0, for\ t > 0$. Use the finite difference scheme to calculate u for 4 time steps with h = 0.25	**4M**
2A.	Solve $(x^3 + 1) y'' + x^2 y' - 4xy = 2$, y(0) = 0, y(2) = 4 with h = 0.5.	**3M**
2B.	Find the m.g.f of the random variable X which is uniformly distributed over (-a, a). Evaluate $E(X^{2n})$	**3M**
2C.	A random sample of size 15 from a population $N(\mu, \sigma^2)$ yields $\overline{X}$ = 3.2 and $S^2 = 4.24$. Find 90% confidence interval for σ^2.	**4M**

3A.	The diameter of an electric cable say 'X' is assumed to be a continuous random variable with the following pdf. $f(x) = \begin{cases} 6x(1-x) & ; \ 0 < x < 1 \\ 0 & ; otherwise \end{cases}$ (i)Obtain an expression for pdf (ii) Find b such that $P(x < b) = 2P(x \geq b)$ (iii) Find $P\left(X < \frac{1}{2} \mid \frac{1}{3} < X < \frac{2}{3}\right)$.	**3M**
3B.	In a normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and variance of the distribution.	***3M***
3C.	Let $\bar{X}$ denote the mean of a random sample size 50 from the distribution χ^2. Compute an approximate value of Pr $\{49 < \bar{X} < 51\}$.	**4M**
4A.	Two independent random variables X_1 and X_2 have mean values 5, 10 and Variance 4, 9. Find covariance between $U = 3X_1 + 4X_2$ and $V = 3X_1 - 5X_2$.	**3M**
4B.	The Mendelian theory states that the probabilities of classification a, b, c,d are respectively $\frac{9}{16}, \frac{3}{16}, \frac{3}{16}, \frac{1}{16}$ from a sample of size 160. The actual observed numbers were 86,35,26,13. Is this data consistent with the theory at 0.05 significance level.	**3M**
4C.	Let $\bar{X}$ and S^2 be the mean and variance of a random sample of size 25 from the distribution $N(3,100)$. Evaluate $i)$ $P(\bar{X} < 6)$ $ii)$ $P(55.2 < S^2 < 145.6)$	4M
5A.	A two dimensional random variable (X, Y) is uniformly distributed over the triangular region R= {(x, y) / 0 < x < y <1}. Find i) P (Y >1) ii) P (X+Y<1/2) (iii) Cov (X,Y)	**5M**
5B.	Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from N (θ_1, θ_2) with usual domain. Find Maximum likelihood parameter (M L E) for θ_1, θ_2	**5M**

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