MANIPAL INSTITUTE OF TECHNOLOGY
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A Constituent Institution of Manipal University

I SEMESTER M.TECH.(ICT) END SEM EXAMINATION

SUBJECT: PROBABILITY AND STOCHASTIC PROCESS (MAT-5157)

Date of Examination: **07-01-2023** Time: **09.30 AM to 12.30 PM** Max. Marks: **50**

<u>Instructions to Candidates:</u> *Answer ALL the questions. Missing data if any may be suitably assumed.*

Q.1.A: Classify the states (recurrent or transient) of the Markov chain whose transition matrix is given by

$$\begin{bmatrix} 1/3 & 2/3 & 0 \\ 1 & 0 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}.$$

Moreover, calculate all the steady state probabilities.

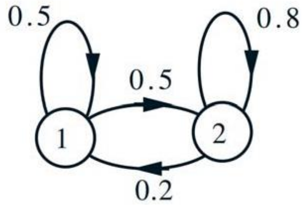
Q.1.B: Let $\{X_n : n \in \mathbb{N}\}$ be a Markov chain with three states 0, 1, 2 and with transition matrix given by

$$\begin{bmatrix} 0 & 3/4 & 1/4 \\ 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$$

Compute the following probabilities assuming that the initial distribution is equally likely for the three states 0, 1 and 2.

i. $P(X_3 = 2 | X_2 = 1)$
ii. $P(X_3 = 1 | X_2 = 2, X_1 = 1, X_0 = 2)$
iii. $P(X_2 = 2)$.

Q.1.C: Consider the following Markov Chain:



Let $r_{ij}(n) = P(X_n = j | X_0 = i)$. Calculate $r_{11}(2)$ and $r_{21}(2)$. (3+3+4=10)

Q.2.A: Three light bulbs have independent exponentially distributed lifetimes with a common parameter λ. Compute the expected value of the time until the last bulb burns out.

Q.2.B: Let $A = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$. Compute A^{2023} given that $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ are eigen vectors of the matrix corresponding to the eigen values 6 and 4, respectively.

Q.2.C: Evaluate the Nash equilibrium for a game which has the following pay-off matrix:

$$\begin{bmatrix} -6, -6 & 0, -10 \\ -10, 0 & -1, -1 \end{bmatrix}.$$

Interpret the game and the equilibrium.

(3+3+4=10)

Q.3.A: A packet consisting of a string of *n* symbols is transmitted over a noisy channel. Each symbol has probability p = 0.0001 of being transmitted in error, independent of errors in the other symbols. How small should *n* be in order for the probability of incorrect transmission (at least one symbol in error) to be less than 0.001?

Q.3.B: Suppose fish are caught as a Poisson Process, $\lambda = 0.6$/hour. You fish for two hours. If no fish is caught within the first two hours, you continue fishing until you catch a fish. Compute the probability of catching at least two fishes. Compute the expected value of total fishing time. Also, compute the expected number of fish caught.

Q.3.C: A coin is tossed till the first head appears. Let T denote the number of tosses. Compute $E(T)$ and $Var(T)$ (Detail calculation is required to receive full marks).

(3+3+4=10)

Q.4.A: In a test has two questions Q1 and Q2. Q1 has three options and Q2 has 4 options. Assume that a student selects one option at random from each of the two questions. Calculate the expected marks.

Q.4.B: The number of items produced in a factory during a particular week has a mean of 500 and variance 100. Prove that the probability that the week's production will lie between 480 and 520 is at least 75%.

Q.4.C: Solve the zero-sum game, where pay-off matrix of the row player is given as follows:

$$\begin{bmatrix} 2 & 0 & 4 \\ 1 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix}.$$

(3+3+4=10)

Q.5.A: Let (X, Y) be a two-dimensional uniformly distributed random vector over the rectangle $[a, b] \times [c, d]$. Compute $E(XY)$ and $E(Y)$.

Q.5.B: Suppose that the quality inspector at the glass manufacturing company inspects 30 randomly selected sheets of glass and records the number of flaws found in each sheet. These data values are shown in the TABLE below. If the distribution of the number of flaws per sheet is taken to have a Poisson distribution, how should the parameter λ of the Poisson distribution be estimated? Calculate the standard error of the estimate.

1	0	1	1	0	0	0	1	1	2
2	1	1	1	0	1	1	1	0	1
2	1	0	1	1	1	0	0	0	2

Q.5.C: Examine if the sum of two independent Poisson random variables is a Poisson random variable.

(3+3+4=10)

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