Exam Date & Time: 06-Jan-2023 (10:00 AM - 01:00 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

Manipal School of Information Sciences (MSIS), Manipal
First Semester Master of Engineering - ME (Big Data Analytics /Artificial Intelligence & Machine Learning) Degree Examination - January 2023

Applied Probability and Statistics [AML 5103]

Marks: 100 **Duration: 180 mins.**

Friday, January 6, 2023

Answer all the questions.

1) [10 points] [TLO 1.1, CO 1] From a group of 21 ES students, 18 AIML students, and 17 BDA students, a group of 5 ES students, 6 AIML students, and 4 BDA students need to be chosen to form a student committee of 15 members; How many different such committees are possible? (10)

2) [10 points] [TLO 1.3, CO 2] Five friends–Aakif, Pranay, Bandavya, Sudeepta, and Megha–decide to meet for lunch at Canara Mall. There are five possible locations in Canara Mall: *(1) Boda Sheera (2) Pizza Hut (3) McDonald's (4) Domino's (5) Chat Shop.* Here is the scenario:

- Aakif is equally likely to go to any of the five places.
- Pranay does not go to the Chat Shop. Otherwise, he is equally likely to go to any of the other four places.
- Bandavya is twice as likely to go to the Chat Shop as any one of the other locations.
- Sudeepta and Megha always go together. They will roll a fair die (with numbers 1 to 6 on its faces) first: if the roll is even, then Sudeepta decides, otherwise Megha will decide where to go. Sudeepta always chooses Domino's and Megha is equally likely to pick any of the five places. (10)

Finally, these four groups will decide on which location to go to independently. Based on the above information, fill the table below row by row. This table will be used in the next problem.

Probability that a particular person or group goes to a particular eat out					
	Boda Sheera	*Pizza Hut*	*McDonald's*	*Domino's*	*Chat Shop*
Aakif					
Pranay					
Bandavya					
Sudeepta & Megha					

3) [10 points] [TLO 1.3, CO 1] We continue with the table that you filled from the previous problem. If you are unsure about an entry in that table, use a letter for that entry and calculate the probability that Bandavya goes to a location and finds none of her friends there. (10)

4) (10)

[10 points] [TLO 1.2, CO 1] A system is comprised of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed, where $i = 1, 2, 3, 4, 5$.

(a) How many outcomes are in the sample space of this experiment?

(b) Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. How many outcomes are contained in the event W?

(c) Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A?

(d) Write out all the outcomes in the event (A and W).

5) [10 points] [TLO 1.3, CO 2] You have tracked the performance of the local meteorologist and compiled the following data:

$$\begin{aligned} &\text{P(forecast rain, and actual rain)} = 0.45, \\ &\text{P(forecast rain, and actual no rain)} = 0.15, \\ &\text{P(forecast no rain, and actual rain)} = 0.05. \end{aligned}$$

(10)

(a) How often does it actually not rain after her forecast?

(b) How often does she make an incorrect forecast?

(c) Given that she had forecast rain in last night's broadcast, what is the probability that it did rain today?

6) [10 points] [TLO 2.1 CO 1] You run APS Mobile Company that offers cell phones with three storage capacities: Small (1GB), Medium (4GB), and Large (8GB). Based on your market research, you believe that the three sizes will be ordered by a potential customer with probabilities 0.2, 0.5, and 0.3, respectively, for each order independently of the other orders. For each of the following, identify the correct random variable with the associated parameters clearly shown. Using the parameters, calculate the expected value of each random variable. Your answer should look like, for example,

$$X \sim \text{Bin}(n = 12, p = 0.2) \text{ and } E[X] = np = 12 \times 0.2.$$

(10)

(a) The number of Medium phone orders out of the next 100 phone orders.

(b) The number of phones that will be ordered until the next Small phone is ordered given that the previous ten phones ordered were not Small.

(c) Suppose that 30 of the previous 100 orders were for Medium phones. You randomly choose 20 of those 100 orders and would like to know the number of phone orders that are not Medium in them.

7) [10 points] [TLO 2.1, CO 1] Suppose that the average number of vehicles abandoned weekly on a certain highway is 2.2. What is the probability that there will be

(a) no abandoned cars in the next week;

(10)

(b) at least 2 abandoned cars in the next two weeks?

8)

(10)

[10 points] [TLO 2.2, CO 1] A typing agency employs 2 typists. The average number of errors per article is 2 when typed by the first typist and 4 when typed by the second typist. Suppose that an article is twice as likely to be typed by the second typist as the first.

(a) Let X and Y be the random variables that represent the number of errors made by the first and second typists, respectively. What type of random variables are X and Y, and what are the associated parameters?

(b) Find the probability that the article will have no errors.

Hint: *start with $P(\text{no error})$, use the law of total probability followed by Bayes' theorem.*

9) [10 points] [TLO 2.2, CO 2] The number of times that a person contracts a cold in a given year is a Poisson random variable with parameter $\lambda = 4$. Suppose that a new wonder drug (based on large quantities of vitamin C) has just been marketed that reduces the Poisson parameter to $\lambda = 1.5$ for 85 percent of the population. For the remaining population, the drug has no appreciable effect on colds. If an individual tries the drug for a year and has 1 cold in that time, we are interested in knowing how likely is it that the drug is beneficial for him or her.

(a) Is the quantity we are trying to calculate:

$$P(\text{Drug beneficial} \mid \text{1 cold})$$
$$\text{or}$$
$$P(\text{1 cold} \mid \text{Drug beneficial})$$
$$\text{or}$$
$$P(\text{1 cold AND Drug beneficial})?$$

(10)

(b) What is the probability that the drug is beneficial?

(c) What is the probability that the drug is not beneficial?

(d) Given that the drug is beneficial, the number of colds $X \sim \text{Poi}(?)$

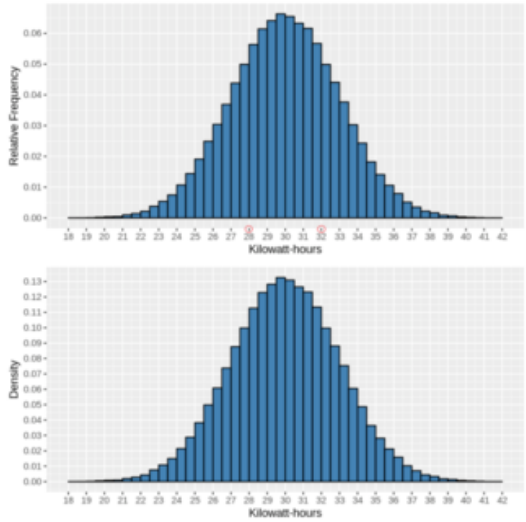
(e) Given that the drug is not beneficial, the number of colds $X \sim \text{Poi}(?)$

(f) Apply Bayes' formula to calculate the probability in part (a).

10)

(10)

[10 points] [TLO 2.3 & 3.1, CO 3] Consider the following histograms corresponding to a random variable that models the electricity consumed (number of kilowatt-hours) by a machine over a day with the 25th and 75th percentiles marked as circles:



Mark the following statements as *true* or *false*. Incorrect answers carry a penalty.

(a) Over the next 1000 days, we can expect the machine to consume around 30 kilowatt-hours on an average per day.

(b) Over the next 1000 days, the machine will consume less than 30 kilowatt-hours per day in approximately 500 days.

(c) The standard deviation of the electricity consumption in the next day is about 5 kilowatt-hours.

(d) There is more than a 20% chance that the machine will consume 26 to 28 kilowatt-hours in the next day.

(e) There is less than a 15% chance that the machine will consume close to 30 kilowatt-hours in the next day.

(f) It is more likely for the machine to consume 38 to 40 kilowatt-hours than 18 to 20 kilowatt-hours in the next day.

(g) There is an approximately 1% chance that the machine will consume less than 18 kilowatt-hours in the next day.

(h) Over the next 1000 days, we can expect approximately 25 days in which the machine will consume 24 to 25 kilowatt-hours.

(i) The inter-quartile range of the electricity consumption in the next day is 8 kilowatt-hours..

(j) There is about an 85% chance that the machine will consume more than 28 kilowatt-hours in the next day.

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