Question Paper

Exam Date & Time: 12-Jul-2023 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SIXTH SEMESTER B.TECH MAKEUP EXAMINATIONS, JULY 2023

GAME THEORY AND APPLICATIONS [ICT 4307]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Two players play a symmetric game where each can either cooperate or defect. If they cooperate, both get an payoff of 1. If they defect, both get a payoff of 2. If one cooperates but the other defects, the one cooperating gets a payoff of 0, and the one defecting a payoff of 3. Write the payoff bimatrix for the given scenario, and find the following: (5)

i) Maxmin moves

ii) Possible domination

iii) Best responses, and

iv) Pure Nash Equilibria.

B) Consider there are two bars on a hill station. Each one of two bars charges its own price for a beer, either ₹20, ₹40, or ₹50. The cost of obtaining and serving the beer can be neglected. It is expected that 6000 beers per month are drunk in a bar by tourists, who choose one of the two bars randomly, and 4000 beers per month are drunk by natives who go to the bar with the lowest price, and split evenly in case both bars offer the same price. What prices would the bars select? (3)

C) (2)

For the game shown in Figure Q.1C, write down the terminal histories, proper subhistories, and information sets.

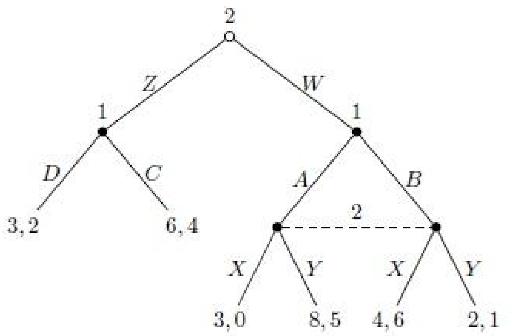

Figure: Q.1C

2) A) For the following matrix game, write the primal and dual LPs and compute all Nash equilibria. (5)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}.$$

B) Suppose that Rose and Colin play the game (3)

		Colin A	Colin B
Rose	A	100, −100	0, 0
	B	20, −20	60, −60

i) What is Colin's optimal strategy in this game?

ii) Show that if Colin plays optimally, Rose's expected outcomes for Rose A and Rose B are lotteries, and Rose is indifferent between these two lotteries.

C) (2)

Consider the correlated strategy for the given game as

$$\alpha = \left((x_1, x_2) = \frac{1}{2}; (x_1, y_2) = 0; (y_1, x_2) = 0; (y_1, y_2) = \frac{1}{2} \right).$$

	x_2	y_2
x_1	2, 2	0, 6
y_1	6, 0	1, 1

Compute the expected utility for each player when α is implemented.

3) A) Consider a 4×2 game whose payoff matrix is given below. (5)

	A	B
A	−3	5
B	−1	3
C	2	−2
D	3	−6

Draw the payoff graph for the given game and solve it in terms of the following:

Draw the payoff graph for the given game and solve it in terms of the following:

i) Value of the game

ii) Rose's (Row player) optimal strategy

iii) Colin's (Column player) optimal strategy.

B) (3)

Consider the Braess paradox, which is usually associated with transportation networks and brings out the counter-intuitive fact that a transportation network with extra capacity added may actually perform worse (in terms of time delays) than when the extra capacity did not exist. Figure Q.3B shows a network that consists of a source S and a destination D, and two intermediate hubs A and B. It is required to travel from S to D. One route is via the hub A and the other route proceeds via the hub B.

Regardless of the number of vehicles on the route, it takes 25 minutes to travel from S to B or from A to D. On the other hand, the travel time from S to A takes time $\frac{m}{50}$ minutes where m is the number of vehicles on that link. Similarly, the travel time from B to D takes time $\frac{m}{50}$ minutes where m is the number of vehicles on that link. Discuss the pure strategy Nash equilibria for the given game.

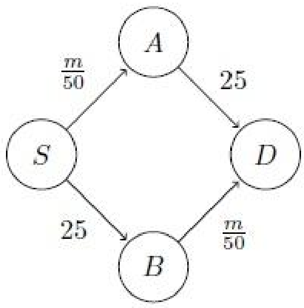

Figure: Q.3B

C) Write the type sets, outcomes and social choice function for supplier selection (assume there are only two suppliers). (2)

4) (5)

A)

Consider the following two player game

		Colin C	Colin D
Rose	A	4, 2	1, 1
Rose	B	1, 1	2, 4

Draw a neat diagram for the convex hull and compute the following for the given game:

i) Nash bargaining solution, and

ii) Simple egalitarian solution.

B) Consider a TU game in characteristic function form $v(1) = v(2) = v(3) = 0, v(12) = 5, v(13) = 2, v(23) = 3, v(123) = 6$. Draw the core in the imputation triangle. What is its shape? List the imputations at its vertices. (3)

C) Consider a TU game (N, v), where $N = \{1, 2, 3\}$ and $v(1) = 3, v(2) = 2, v(3) = 1; v(12) = 8, v(13) = 6.5, v(23) = 8.2, v(123) = 11.2$. (2)

i) Is this game superadditive?

ii) Is it convex?

5) A) Consider that the Parliament of a certain Nation has four political parties A, B, C and D with 45, 25, 15, and 12 members respectively. To pass any bill, at least 51 votes are required. (5)

i) Model the given scenario as TU game in characteristic function form, and

ii) Perform Shapley-Shubic power analysis for the given scenario.

B) (3)

For a general 3-person game in 0-normalized form

$$v(1) = v(2) = v(3) = 0$$

$$v(12) = a \quad v(13) = b \quad v(23) = c$$

$$v(123) = d.$$

Show that the Shapley values are given by

$$\varphi_1 = \frac{a + b + 2(d - c)}{6} \quad \varphi_2 = \frac{a + c + 2(d - b)}{6} \quad \varphi_3 = \frac{b + c + 2(d - a)}{6}.$$

C) (2)

Consider the following problem, plot the feasible region and circle all the corner point feasible (CPF) solutions. (2)

$$\begin{aligned} \text{maximize} \quad & Z = x_1 + 2x_2 \\ \text{subject to} \quad & x_1 \leq 2, \\ & x_2 \leq 2, \\ & x_1 + x_2 \leq 3, \\ & x_1, x_2 \geq 0 \end{aligned}$$

Also, write given LP in augmented form.

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