Question Paper

Exam Date & Time: 03-Jun-2023 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

VI SEMESTER B.TECH
END SEMESTER EXAMINATIONS
JUNE 2023

GAME THEORY AND APPLICATIONS [ICT 4307]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates:
1. Answer ALL questions
2. Missing data may be suitably assumed and clearly stated

1) A) Amit and Beth are not on speaking terms, but have a lot of common friends. Both want to invite them to a dinner party this weekend, either Friday or Saturday evening. Both slightly prefer Saturday. If both set the party at the same time, this will be considered a disaster with a payoff of -10 for both. If one plans the party on Friday and the other on Saturday, the one having the Saturday party gets a payoff of 5, and the other of 4. Write the payoff bimatrix for the given scenario, and find the following: (5)

i) Maxmin moves
ii) Domination
iii) Best response, and
iv) Pure Nash Equilibria.

B) (3)

Consider the given bimatrix for a two-player game. The mixed strategy profile for player 1 and 2 are $\sigma_1 = [0.2\ 0.5\ 0.3]^T$, and $\sigma_2 = [0.4\ 0.1\ 0.5]^T$ respectively.

	D	E	F
A	$1,1$	$3,4$	$2,1$
B	$2,4$	$2,5$	$8,1$
C	$3,3$	$0,4$	$0,9$

i) Compute the payoffs $u_1(\sigma_1, \sigma_2)$, and $u_2(\sigma_1, \sigma_2)$
ii) Find the support of mixed strategy profile.

C) For the game shown in Figure 3, write down the terminal histories, proper sub-histories, and information sets. (2)

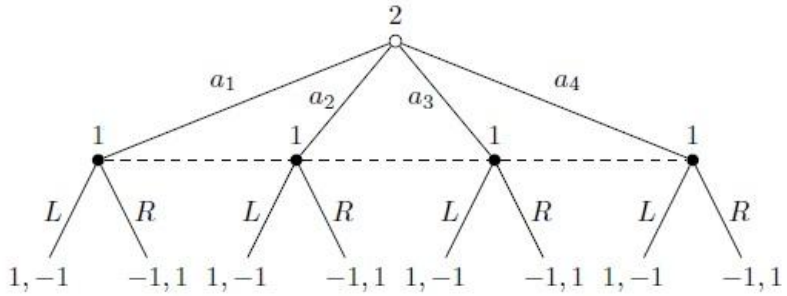

Figure: 3

2) (5)

A)

Consider the scenario of *Tragedy of Commons*, which represents a type of social paradox or social tragedy. The problem involves a conflict over use of resources between individual interests and social interests. A village has n farmers represented by the set $N = \{1, 2, \ldots, n\}$. Each farmer has the option of keeping a sheep or not. If 1 corresponds to keeping a sheep and 0 corresponds to not keeping a sheep, the strategy sets are given by

$$S_1 = S_2 = \cdots = S_n = \{0, 1\}.$$

The utility from keeping a sheep (that arises because of milk, wool, etc.) is equal to 1 unit. The village has a limited stretch of grassland and when a sheep grazes on this, there is a damage to the environment, equal to 5 units. This damage to the environment is to be shared equally by the farmers. Write the payoff for ith farmer and find the dominated strategies for the following cases:

i) $n < 5$

ii) $n = 5$

iii) $n > 5$

iv) $n > 5$ when the government decides to impose a tax of 5 units for each kept sheep.

B) Check whether the given game is equivalent to a zero-sum game. If so find a positive linear transformation of one of the player's utilities which shows the equivalence. If not say how you know that. (3)

A B

A	$0, 10$	$1, -10$
B	$3, -50$	$-1, 30$

C) (2)

Consider the following two-player game.

	A	B	C
A	$10, 10$	$14, 12$	$14, 15$
B	$12, 14$	$20, 20$	$28, 15$
C	$15, 14$	$15, 28$	$25, 25$

Find the matrix obtained by IESD.

3) Consider a 2×5 game whose payoff matrix is given below. (5)

A)

	A	B	C	D	E
A	-2	5	1	0	-4
B	3	-3	-1	3	8

Use graphical method to solve this game in terms of following:

i) Value of the game

ii) Rose's (Row player) optimal strategy

iii) Colin's (Column player) optimal strategy.

B) Linear programs enable us to compute mixed strategy equilibria. For the given matrix game, write the linear programs for row and column player. (3)

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$$

C) (2)

If Rose's utilities for v, w, x, and u are 0, 20, 60, and 100, respectively, say which alternative Rose would prefer in each of the pairs:

i) x vs. $\frac{3}{4}w, \frac{1}{4}u$

ii) x vs. $\frac{1}{2}w, \frac{1}{2}u$

iii) $\frac{1}{2}w, \frac{1}{2}x$ vs. $\frac{1}{2}v, \frac{1}{2}u$

iv) $\frac{3}{7}w, \frac{4}{7}x$ vs. $\frac{3}{7}u, \frac{4}{7}v$.

4) (5)

A) Consider a two person bargaining problem (F, v), where F is the convex hull enclosing the points $\{(1,8),(6,7),(8,6),(9,5),(10,3),(11,-1),(-1,-1)\}$ and $(2,1)$ is the default value, v. Draw a neat diagram for the convex hull and compute the following for the given scenario:

i) Nash bargaining solution, and

ii) Simple egalitarian solution.

B) (3)

Consider the following TU game in characteristic function form

$$v(A) = 1; v(B) = 2; v(C) = 3; v(AB) = 8.2; v(AC) = 6.5; v(BC) = 8; v(ABC) = 11.2.$$

Normalize the given TU game and compute the core.

C) (2)

Consider a TU game (N, v), where $N = \{1,2,3\}$ and $v(1) = v(2) = 1; v(3) = 2; v(12) = v(23) = v(13) = 4; v(123) = 5$.

i) Is this game superadditive?

ii) Is it convex?

5) (5)

A)

Consider there is a music band in MIT, Manipal comprising of a singer, a pianist and a drummer. The band performs at various institution of MAHE and receives honorarium for their performance. Assume that for one such performance the hosting institute promises ₹10,000. While performing individually or duets the members can earn as follows:

- Singer-Pianist: ₹8000
- Drummer-Pianist: ₹6500
- Singer-Drummer: ₹5000
- Pianist: ₹3000
- Singer: ₹2000
- Drummer alone cannot earn anything.

i) Consider the given scenario as a TU game and express it in characteristic function form.

ii) Divide the honorarium received from the host institution among the band members based on Shapley value computation.

B) Consider the voting system [16: 7, 6, 3, 3, 2]. Find the Banzhaf power index. (3)

C) Write the type sets, outcomes and social choice function in procurement of an indivisible object from two sellers. (2)

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