Question Paper

Exam Date & Time: 12-Jun-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - MAY 2023
II SEMESTER B.Sc (Applied Sciences) in Engg.

MATHEMATICS - II [IMA 121 - S2]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

Missing data, if any, may be suitably assumed

1)

A) Evaluate $\iint \frac{dxdy}{x^4+y^2}$ over the region $y \geq x^2, x \geq 1$. (3)

B) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}} dxdy$ by changing to polar co-ordinate system (3)

C) Find the volume of the region enclosed by the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $z = x^2 + y^2$ using triple integration. (4)

2)

A) Find the equation of the sphere which touches the plane $x - 2y - 2z = 7$ at the point $L(3, -1, -1)$ and passes through the point $M(1, 1, -3)$ (3)

B) Find the equation of the right circular cylinder having the circle $x^2 + y^2 + z^2 = 9, x - y + z = 3$ as a base circle. (3)

C) Show that $\int_0^\infty \sqrt{y} e^{-y^2} dy \; X \; \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}} dy = \frac{\pi}{2\sqrt{2}}$ (4)

3)

A) Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point (1, -2, 1) (3)

B) Verify Green's theorem in the plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where c is the boundary of the region defined by $y = \sqrt{x},\ y = x^2$ (3)

C) Find all the eigen values and any one corresponding eigen vector of the matrix (4)

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

4)

A) Solve the following using Gauss-Seidel method. Carry out 4 iterations. (3)

$$10x - 2y - 2z = 6$$
$$-x + 10y - 2z = 7$$
$$-x - y + 10z = 8$$

B) (3)

Solve the following using Gauss Jordan method.

$-3x + 2y + 2z = 8$

$x + 4y - 6z = 1$

$-2y + 2z = -2$

C) (4)

Construct orthonormal basis from the following set of vectors

$\{(1, -1, 0), (2, -1, -2), (1, -1, -2)\}$

5) (3)

Any minimal spanning set of vectors forms a basis

A)

B) (3)

Prove that the set $A = \{(1,1,1), (1,1,0), (1, 0, 0)\}$ forms a basis for R^3.

C) (4)

Show that $\vec{F} = (2xy + z^3)i + x^2 j + 3xz^2 k$ is conservative field. Hence find the scalar potential.

-----End-----