MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

II SEMESTER M. TECH (CSE/CSIS: Program Elective I)

END SEMESTER MAKEUP EXAMINATION, 04-JULY-2023

SUBJECT: FUNDAMENTALS OF QUANTUM COMPUTING (CSE 5025)

REVISED CREDIT SYSTEM

Time: 3 Hours **MAX. MARKS: 50**

Note: Answer ALL the questions.

1A	Let $\|\psi_1\rangle = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ and $\|\psi_2\rangle = \left(\frac{1}{\sqrt{3}}\|0\rangle + \sqrt{\frac{2}{3}}\|1\rangle\right)$. Compute $\langle\psi_1 \| \quad \psi_2\rangle$.	02
1B	Consider the following quantum state $\|\psi\rangle = \frac{1}{2}(\|00\rangle - \|10\rangle + \|01\rangle - \|11\rangle)$ The first qubit is measured. What is the probability that the result is 0? What is the probability that the result is 1? For each possible result, write down the post-measurement state, and calculate the probability that a measurement of the second qubit will give 0 and 1. Write down the states after the second measurement.	05
1C	Imagine we can define a unitary operator U that can copy the qubit states $\|\psi_1\rangle = \frac{1}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ and $\|\psi_2\rangle = \frac{1}{\sqrt{2}}(\|0\rangle - \|1\rangle)$: $U\|\psi_1\rangle\|0\rangle = \|\psi_1\rangle\|\psi_1\rangle$; $U\|\psi_2\rangle\|0\rangle = \|\psi_2\rangle\|\psi_2\rangle$. Can U be used to copy $\|1\rangle$? Verify using an explicit calculation.	03
2A	Let $\|\psi_1\rangle = \left(\frac{1}{\sqrt{3}}\|0\rangle + \sqrt{\frac{2}{3}}\|1\rangle\right)$ and $\|\psi_2\rangle = \frac{1}{2}\|0\rangle + \frac{\sqrt{3}}{2}\|1\rangle$. Compute $\|\psi_1\rangle \otimes \|\psi_2\rangle$.	03
2B	Alice wishes to exchange a private encryption key with Bob using the BB84 protocol. She generates a binary string 0110100010010111, and encode using H H H I I H I H H I I I H I I H . What is the quantum state that she transmits to Bob? Alice sends the state to Bob, who decodes using I H H I H I H H I H I I I H I H. They both reveal their choice of encoding. What is the string they retain?	05
2C	Design quantum circuit to implement Bell state $\frac{1}{\sqrt{2}}(\|01\rangle + \|10\rangle)$.	02
3A	With quantum circuit implement AND, OR and NOT gates using CCNOT gate.	05
3B	Define Fredkin gate. Design quantum circuit for Fredkin gate and compute outputs for all possible inputs. Represent Fredkin gate in matrix form.	03
3C	Suppose a two-qubit system is in the state $\frac{1}{\sqrt{2}}\|00\rangle + \frac{1}{\sqrt{2}}\|11\rangle$. A Pauli X gate (i.e. a NOT gate) is applied to the second qubit, and a Hadamard gate is applied to the first qubit. (i) What is the new state of the system? (ii) What are the probabilities of the possible outcomes if both qubits are now measured	02

4A For the following combinations of a, N, apply Shor's algorithm to find the factors of N. 05
N = 15, a = 11

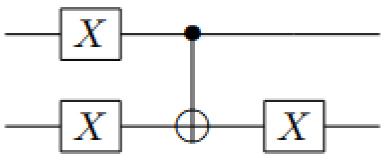
4B Suppose that Charlie intercepts the qubit transmitted by Alice in the Superdense coding protocol. Can she infer which of the four pairs of bits 00; 01; 10, or 11 Alice was trying to transmit? If so, how? If not, why not? 02

4C Show that the following two qubit quantum state is an entangled state. 03

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

5A Apply Grover's algorithm on a system with N= 4 and solution is indexed by x = 0. 03

5B Compute outputs for all inputs $|00\rangle, |01\rangle, |10\rangle, and\ |11\rangle$ for the following 2 qubit quantum circuit and give its matrix representation.



04

5C Design quantum circuit for encoding the state $|\psi\rangle = a|0\rangle + b|1\rangle$ using Shor's 9 qubit code. 03