Exam Date & Time: 04-Jan-2024 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FIRST SEMESTER B.TECH. EXAMINATIONS - JANUARY 2024
SUBJECT: MAT 1171/ MAT_1171 - ENGINEERING MATHEMATICS - I

Engineering Mathematics - 1 [MAT-1171]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

1A) Using Lagrange's interpolation formula, find a polynomial $y = f(x)$ from the following data (4)

x	0	1	3	4
y	-12	0	6	12

1B) Using Newton Raphson method, find the real root of the equation $x^3 = 6x - 1$, by taking the initial approximate root as $x_0 = 0.5$, carry out three iterations and correct up to 4 decimal places. (3)

1C) From the following table of values of x and y, find $\frac{dy}{dx}$ at $x = 0.8$, correct to four decimal places. (3)

x	0.4	0.5	0.6	0.7	0.8
y	1.5836	1.7974	2.0442	2.3275	2.6510

2A) Using Runge Kutta method of order 4, find $y(0.1)$ from the given equation $\frac{dy}{dx} = \frac{x-y}{2}$, with $y(0) = 1$. Take $h = 0.1$ and correct to 4 decimal places. (4)

2B) Using Taylor series method, find an approximate value of y when $x = 0.1$ given that $\frac{dy}{dx} = x - y^2$; $y(0) = 1$, carryout upto fourth order derivative terms in the series, correct to 4 decimal places. (3)

2C) Using Simpson's $\frac{1}{3}^{rd}$ rule to find an approximate value for the integral $\int_0^{0.6} e^{-x^2} dx$ by taking 6 subintervals and correct to 4 decimal places. (3)

3A) Using Gram-Schmidt orthogonalization process find an orthonormal basis of $\mathbb{R}^3$ from the set of vectors $\{(1,-1,1), (1,0,1), (1,1,2)\}$. (4)

3B) Solve $(2xy - \sin x)dx + (x^2 - \cos y)dy = 0$ (3)

3C) Solve $\frac{d^2y}{dx^2} + 3\ \frac{dy}{dx} + 2y = x^2\ + 3x\ +\ 1.$ (3)

4A) Show that the vectors $S = \{(1,1,1),(1,2,3),(2,-1,1)\}$ form a basis for $\mathbb{R}^3$. Express the vector $(2,5,4)$ in terms of basis vectors. (4)

4B) Using method of variation of parameters, evaluate

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x \log x.$$ (3)

4C) Using Gauss elimination method, solve the system equations

$$x + y + z = 6$$

$$x - y + 2z = 5$$ (3)

$$3x + y + z = 8$$

5A) Using power method, obtain the numerically largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 3 & 4 & -2 \\ 1 & 4 & -1 \\ 2 & 6 & -1 \end{bmatrix}$ correct to 2 decimals, after 4 iterations. Take initial approximate eigen vector as $X^{(0)} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. (4)

5B) Solve $x^2\ \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 5y = x^2 \sin\,(\ln x).$ (3)

5C) Using Gauss Seidel method solve the following system of equations,

$$10x + 2y + z = 9$$

$$-2x + 3y + 10z = 22$$ (3)

$$x + 10y - z = -22$$

Perform 3 iterations and correct the solution up to 4 decimals by taking the initial approximation as $x = y = z = 0.$

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