Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. (BIO+CHE)

END SEMESTER MAKE-UP EXAMINATIONS, JANUARY 2024

ENGINEERING MATHEMATICS III [MAT2124]

REVISED CREDIT SYSTEM

Time: 3 Hours **Date:** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

Q.NO.	Questions	Marks	CO	BTL
1A.	Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t}\cos x$ using direct integration, given that $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ when $x = 0$.	(4)	5	3
1B.	Express $f(x) = x$ as a half-range cosine series in $0 < x < 2$.	(3)	4	3
1C.	Obtain the first harmonic of the Fourier series for $f(x)$, where $f(x)$ is given in the following table: x: 0, $\pi/3$, $2\pi/3$, π, $4\pi/3$, $5\pi/3$ $f(x)$: 1.0, 1.4, 1.9, 1.7, 1.5, 1.2	(3)	4	3
2A.	Suppose $$\vec{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}.$$ Evaluate $\int_C \vec{F}.d\vec{r}$ from $(0,0,0)$ to $(1,1,1)$ along the path $C: x = t, y = t^2, z = t^3$.	(4)	4	3
2B.	Find equations for the tangent plane and normal line to the surface $x^2yz - 4xyz^2 + 6 = 0$ at the point $(1,2,1)$.	(3)	4	3
2C.	Expand $f(z) = \frac{1}{z(1-z)}$ in a Laurent series valid for $0 < \lvert z \rvert < 1$. Hence using the same Laurent series, find the residue of $f(z)$ at $z = 0$.	(3)	2	3
3A.	Using Cauchy's integral formula for derivatives, compute $\oint_C \frac{z+1}{z^4+2iz^3}dz$, where C is the circle $\lvert z \rvert = 1.5$.	(4)	2	3
3B.	Evaluate $\oint_C \frac{1}{(z-1)^2(z-3)}dz$, where C is the circle $\lvert z \rvert = 2$.	(3)	3	3
3C.	For $f(z) = \frac{\cos z}{z^2(z-\pi)^2}$, examine the points of singularities and their nature and, hence, find the residues.	(3)	3	3

4A.	If $f(z)$ is analytic, then show that $$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert f(z)\rvert^2 = 4\lvert f'(z)\rvert^2.$$	(4)	3	3
4B.	Suppose $S(z) = (z-i)/(iz-1)$ and $T(z) = (2z-1)/(z+2)$. Find $S^{-1}(T(z))$.	(3)	4	3
4C.	Find the flux of $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ through the sphere of radius 4 and center at the origin. (Take the unit normal vector $\hat{n}$ pointing outward.)	(3)	1	3
5A.	Solve the equation $u_{xy} - u_{yy} = 0$ using the transformation $v = x$ and $z = x + y$.	(3)	1	3
5B.	Suppose $\rho(x,y,z) = 3x^2y + y^2z^2$. Find $\nabla\rho$ (or grad ρ) at the point (1, -2, -1).	(2)	4	3
5C.	Assuming the most general solution, find the temperature $u(x,t)$ in a laterally insulated bar of length $10cms$, whose ends are kept at zero-degree centigrade, and the initial temperature is given by $u(x,0) = x(10-x),\ 0 < x < 10$.	(5)	5	3