Reg. No.

MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL

(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. (BIO+CHE)

END SEMESTER EXAMINATIONS, DEC 2023

ENGINEERING MATHEMATICS III [MAT2124]

REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 07 Dec 2023** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

Q.NO	Questions	*Marks*	*CO*	*BTL*
1A.	(i) Solve $u_{xx} - 2u_{xy} + u_{yy} = 0$ using the transformation $v = x$ and $x = z + y$. (ii) Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$ using direct integration, given that $u = 0$ when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ when $x = 0$.	*(4)*	*5*	*3*
1B.	Find equations for the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point $(2, -1, 5)$.	*(3)*	*4*	*3*
1C.	Suppose a force field is given by $\vec{F} = (2x - y + z)\hat{\imath} + (x + y - z^2)\hat{\jmath} + (3x - 2y + 4z)\hat{k}$. Find the *work done* in moving a particle once around a circle *C* in the $xy -$ plane with its center at the origin and a radius of 3.	*(3)*	*4*	*3*
2A.	(i) Use divergence theorem to evaluate $\iint_S \vec{F}.d\vec{S}$, where $\vec{F} = (x^2 - yz)\hat{\imath} + (y^2 - zx)\hat{\jmath} + (z^2 - xy)\hat{k}$, and S is the surface of the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0,\ \& \ z = c$. (ii) Use Stokes' theorem to find the flux of $\nabla \times \vec{F}$ through S, where S is the upper half surface of the unit sphere centred at origin, and $\vec{F} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$.	*(4)*	*4*	*3*
2B.	Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.	*(3)*	*4*	*3*
2C.	Determine where the following complex mappings $w = f(z)$ are conformal: (i) $f(z) = sin\ z$ (ii) $f(z) = z^3 - 3z + 1$	*(3)*	*2*	*3*

<table>
<tr><td></td><td>(iii) $f(z) = ze^{z^2-2}$.</td><td></td><td></td><td></td></tr>
<tr><td>3A.</td><td>(i) Compute the analytic function $f = u + iv$ where $u(x, y) = x + e^{-x}\cos y$.
(ii) Show that $f(z) = x + i4y$ is nowhere analytic on complex plane.</td><td><i>(4)</i></td><td><i>2</i></td><td><i>3</i></td></tr>
<tr><td>3B.</td><td>Find the Laurent's series expansion of $f(z) = \frac{1}{z-1} + \frac{3}{(z-1)(z-3)}$ that is valid when $0 < |z-1| < 2$. Hence use Laurent's series expansion of f to obtain the residue of f at 1.</td><td><i>(3)</i></td><td><i>3</i></td><td><i>3</i></td></tr>
<tr><td>3C.</td><td>Using Cauchy's Residue theorem, determine the value of integral-
$$\int_C \frac{1}{z^3(z+4)}dz$$
where C is the positively oriented circle $|z+2| = 3$.</td><td><i>(3)</i></td><td><i>3</i></td><td><i>3</i></td></tr>
<tr><td>4A.</td><td>(i) Let $f(z) = \frac{\cos z}{z^2(z-\pi)^2}$. Examine the points of singularities and their nature and, hence, find the residues for $f(z)$.
(ii) Evaluate $\oint_C \frac{dz}{z^2+1}$, where $C: |z-1| = 1$.</td><td><i>(4)</i></td><td><i>3</i></td><td><i>3</i></td></tr>
<tr><td>4B.</td><td>WITHOUT using the Divergence Theorem find the flux of $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ through the sphere of radius 4 and center at the origin. (Take the unit normal vector $\hat{n}$ pointing outward.)</td><td><i>(3)</i></td><td><i>4</i></td><td><i>3</i></td></tr>
<tr><td>4C.</td><td>Obtain the first two coefficients in the Fourier series for y, where y is given in the following table:
<table>
<tr><td>x</td><td>0</td><td>π/3</td><td>2π/3</td><td>π</td><td>4π/3</td><td>5π/3</td><td>2π</td></tr>
<tr><td>y</td><td>1.0</td><td>1.4</td><td>1.9</td><td>1.7</td><td>1.5</td><td>1.2</td><td>1.0</td></tr>
</table></td><td><i>(3)</i></td><td><i>1</i></td><td><i>3</i></td></tr>
<tr><td>5A.</td><td>Expand the following function as a Fourier Series $f(x) = x\,sin\,x, -\pi < x < \pi,\ f(x+2\pi) = f(x)$.</td><td><i>(3)</i></td><td><i>1</i></td><td><i>3</i></td></tr>
<tr><td>5B.</td><td>Determine the constant a so that the following vector field is solenoidal:
$\vec{V} = (-4x - 6y + 3z)\hat{\imath} + (-2x + y - 5z)\hat{\jmath} + (5x + 6y + az)\hat{k}$.</td><td><i>(2)</i></td><td><i>4</i></td><td><i>3</i></td></tr>
<tr><td>5C.</td><td>Assuming the most general solution, find the temperature $u(x,t)$ in a laterally insulated bar of length $10cms$, whose ends are kept at zero-degree centigrade, and the initial temperature is given by $u(x,0) = x(10-x),\ 0 < x < 10$.</td><td><i>(5)</i></td><td><i>5</i></td><td><i>3</i></td></tr>
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