Exam Date & Time: 07-Dec-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

THIRD SEMESTER B.TECH END SEMESTER EXAMINATIONS, DEC 2023

ENGINEERING MATHEMATICS-III [MAT 2125]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates: Answer ALL questions Missing data may be suitably assumed

1) A) Find the Fourier series expansion of $f(x) = x(2\pi - x)$ in $0 \le x \le 2\pi$. (4)

B) Find the half range sine series for the function $f(x) = \frac{1}{2} - x,$ in $0 \le x \le 1$. (3)

C) An insurance company insured 2000 scooter drivers, 4000 car drivers, and 6000 truck drivers. The probability of an accident involving a scooter driver, car driver, and a truck is 0.01, 0.03, and 0.015 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? (3)

2) A) Find the Fourier Transform of $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a > 0 \end{cases}$. Hence deduce that $\int_0^\infty \left(\frac{sint}{t}\right)^2 dt = \frac{\pi}{2}$ using Parseval's identity. (4)

B) 'A'can hit a target 3 times in 5 shots, 'B' 2 times in 5 shots and `C' 3 times in 4 shots. They fire a volley. What is the probability that
(i) two shots hit
(ii) atleast two shots hit? (3)

C) A bag contains 19 tickets, numbered from 1 to 19. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers. (3)

3) A) A two-dimensional random variable has joint pdf (4)

$$f(x) = \begin{cases} x^2 + \frac{xy}{3}, 0 < x < 1, 0 < y < 2 \\ 0 \quad otherwise \end{cases}$$

. Compute $E(X)$ and $V(Y)$.

B) A random variable X has the following probability function.

x	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

(3)

(i) Find the value of k.

(ii) Evaluate $P(X < 6)$

(iii) Evaluate P(0 < X < 5)

C) Find the directional derivative of the equation $\emptyset = x^2yz + 4xz^2$ at $(1, -2, -1)$ along $2i - j - 2k$. (3)

4) If $\vec{r} = xi + yj + zk,$ then prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. (4)

A)

B) Evaluate $\iint_S \; f.n\, ds$ where $f = (x+y)i - 2yj + zk$ and S is the surface of the plane $2x + y + 2z = 6$ in the first octant. (3)

C) Solve $u_{xx} + 2u_{xy} + u_{yy} = 0,$ using the transformation $v = x, z = x - y.$ (3)

5)

A) Verify Green's theorem for the function $f = (x^2 + y^2)i - 2xyj$ taken around a rectangle in the XY plane bounded by $x = 0, x = a, y = 0, y = b$. (4)

B) Derive one dimensional wave equation with necessary assumptions. (3)

C) (3)

Solve the partial differential equation $\frac{\partial^2 u}{\partial x^2} = x^2 + y^2$ by direct integration

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