Exam Date & Time: 09-Jan-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FIFTH SEMESTER B.TECH END SEMESTER MAKEUP EXAMINATIONS, JAN 2024

MATHEMATICAL FOUNDATIONS FOR DATA SCIENCE-III [MAT 3151]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates:
Missing data may be suitably assumed

1) If G is a graph with n vertices and minimum degree $\delta(G) \geq \frac{n-1}{2}$, then prove that G is connected. Give an example for a graph on 7 vertices with minimum degree 3. (3)

A)

B) If $diameter(G) \geq 3,$ then show that $diameter(\overline{G}) \leq 3.$ Hence, show that every non trivial self complementary graph has diameter 2 or 3. (3)

C) Let G be labelled graph with A(G) or A as its adjacency matrix. Prove that the $(i,j)^{th}$ entry of A^k is the number of walks of length k from the vertices v_i to v_k. Wrire the adjacency matrix A of complete graph K_5 and find the $(1,5)^{th}$ entry of A^3. (4)

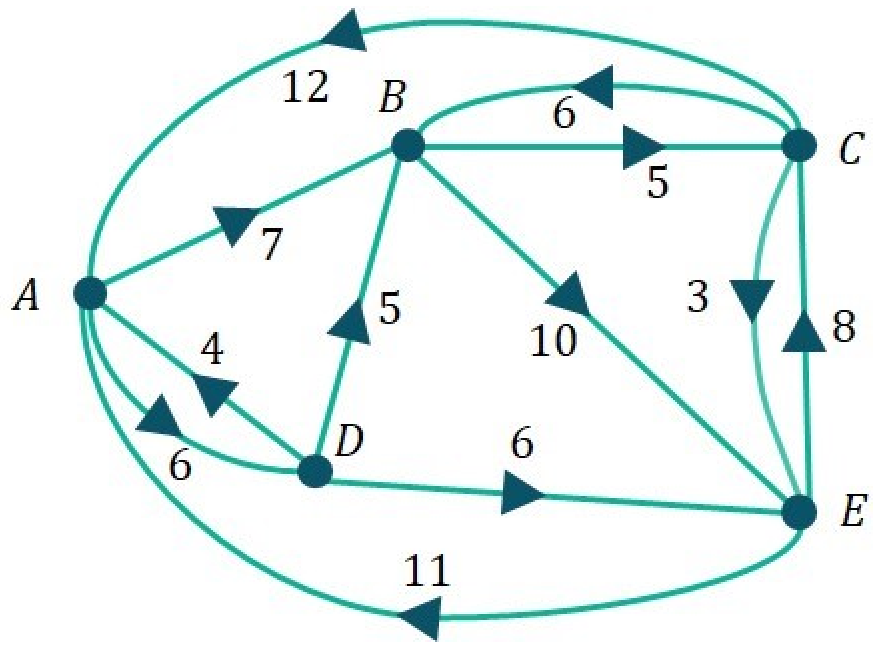
2) Prove that a tree on n vertices has n-1 edges. Draw a tree with 8 vertices having exactly 2 pendant vertices. (3)

A)

B) Define betweenness centrality. Obtain the betweenness centrality of all the vertices in

(i) a cycle graph on 6 vertices and (ii) a complete bipartite graph, $K_{1,n-1}$. (3)

C) Using Dijktra’s algorithm, obtain the shortest path from the vertex B to every other vertices for the graph as shown below. (4)



3) A) Obtain the determinant of adjacency matrix of (i) a cycle graph on 8 vertices (ii) a path graph on n vertices (iii) a graph G which is obtained by removing one edge from a complete graph on 4 vertices, by obtaining all the elementary spanning subgraphs and using the formula $\det(A(G)) = \sum(-1)^{n-c_1(H)-c(H)}\, 2^{c(H)}$, where summation runs over all elementary spanning subgraphs H of G and c_1(H) and c(H) are the number of components which are K_2’s and cycles respectively. (3)

B) Find the highest power of 5 dividing 65! . (3)

C) i) Find the discriminant of f(x,y) =$13x^2 + 17xy + 19y^2$.

ii) Give an example of a binary quadratic form f(x,y) with the discriminant d = 8. (4)

4) Compute the day for the date April 1, 2003; using the formula

A) $d \equiv N + [2.6M - 0.2] + Y + \left[\frac{Y}{4}\right] + \left[\frac{C}{4}\right] - 2C - (1 + L)\left[\frac{M}{11}\right]$ (mod 7). (3)

B) Find the remainder when 15! is divided by 17. (3)

C) Encipher the message "HAVE A NICE TRIP" using a Vigenère cipher with the keyword "MATH". (4)

5) Find the remainder when 444^{44} divided by 7. (3)

A)

B) Find the number of integers in the set $S = \{1,2,3, \ldots, 1800\}$ that are divisible either by 3 or 5. (3)

C) i) Compute the Jacobi symbol $\left(\frac{25}{77}\right)$.

ii) Compute the Legendre symbol $\left(\frac{9}{13}\right)$. (4)

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