MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

III SEMESTER B.TECH. END SEMESTER EXAMINATIONS, DECEMBER- 2023

SUBJECT: ENGINEERING MATHEMATICS III _ MAT 2121

Branch: Mech/IE/MT/Aero/Auto

Time: 3 Hours (05/12/2023) MAX. MARKS: 50

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data may be suitably assumed.

Q. No	Question	M	CO	Blooms Taxo
1A	Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0, u(x,0) = (1 - x^3)$, $u(0,t) = (1 - t^3), u(1,t) = 0$ *and* $\frac{\partial u}{\partial t}(x,0) = 50x$. Compute u for 3 time levels by taking $h = \frac{1}{4}$.	4	CO2	4
1B	Solve $u_{xx} + u_{yy} = 0, 0 < x < 1,\ 0 < y < 1$ subjected to $u(x,1) = x^3 - 3,\ u(0,y) = y^3 - 3,\ u(1,y) = 4,\ u(x,0) = 2$ with $h = \frac{1}{3}$.	3	CO2	3
1C	Use Crank Nicolson's method to solve $\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2}$, $0 < x < 1, t > 0,\ u(x,0) = 0,\ u(0,t) = u(1,t) = 100t$. Choosing $h = \frac{1}{4}$ *and* $\lambda = 1$, compute u for one time step.	3	CO2	3
2A	Find the Fourier series expansion of $f(x) = x^2$, in the interval $(0, 2\pi)$ where $f(x + 2\pi) = f(x)$.	4	CO3	4
2B	Solve $y'' = y + x(x - 4)$ with $y(0) = 0,\ y(4) = 0$ and $h = 1$, using finite difference method.	3	CO1	3
2C	Find the half range sine series of $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 < x < 2 \end{cases}$ Hence Deduce $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$	3	CO3	3
3A	Show that $\vec{F} = (6xy + z^3)\,\hat{\imath} + (3x^2 - z)\hat{\jmath} + (3xz^2 - y)\hat{k}$ is irrotational. Find φ such that $\vec{F} = \nabla\varphi$.	4	CO4	4

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<tr><td>3B</td><td>Find the Fourier transform of $f(x) = \begin{cases} 1-|x| & if\ |x| \le 1 \\ 0 & if\ |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt$.</td><td>3</td><td>CO3</td><td>4</td></tr>
<tr><td>3C</td><td>Find the angle between the surfaces $z = x^2 + y^2$ and $z = \left(x - \frac{1}{\sqrt{6}}\right)^2 + \left(y - \frac{1}{\sqrt{6}}\right)^2$ at the point $\left(\frac{1}{2\sqrt{6}}, \frac{1}{2\sqrt{6}}, \frac{1}{12}\right)$.</td><td>3</td><td>CO4</td><td>2</td></tr>
<tr><td>4A</td><td>Verify Green’s theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where the closed curve of the region bounded by $y = x,\ \ y = x^2$.</td><td>4</td><td>CO4</td><td>4</td></tr>
<tr><td>4B</td><td>The following table gives the variation of a periodic current ‘A’ over a period T
<table>
<tr><td>t (sec.)</td><td>0</td><td>T/6</td><td>T/3</td><td>T/2</td><td>2T/3</td><td>5T/6</td><td>T</td></tr>
<tr><td>A (amp.)</td><td>1.98</td><td>1.30</td><td>1.05</td><td>1.30</td><td>−0.88</td><td>−0.25</td><td>1.98</td></tr>
</table>
Show that there is a constant part of 0.75 Amp in the current A and obtain the amplitude of the first harmonic.</td><td>3</td><td>CO3</td><td>2</td></tr>
<tr><td>4C</td><td>Find the work done in moving a particle in the force field $\vec{F} = 3x^2 i + (2xz - y)j + zk$ along the line from (0,0,0) to (2,1,3).</td><td>3</td><td>CO4</td><td>3</td></tr>
<tr><td>5A</td><td>Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ using the transformation $v = x + ct, w = x - ct$ subject to the conditions $u(x,0) = \varphi(x)$ and $u_t(x,0) = 0$</td><td>4</td><td>CO5</td><td>4</td></tr>
<tr><td>5B</td><td>Solve the partial differential equation $\frac{\partial^3 z}{\partial x \partial y^2} + xy + \sin(2x - 3y) = 0$ by direct integration.</td><td>3</td><td>CO5</td><td>2</td></tr>
<tr><td>5C</td><td>Using Stoke’s theorem, evaluate $\int_C (x+y)dx + (2x - z)dy + (y+z)dz$ where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0)$ and $(0,0,6)$.</td><td>3</td><td>CO4</td><td>4</td></tr>
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