Exam Date & Time: 04-Dec-2023 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FIFTH SEMESTER B.TECH END SEMESTER EXAMINATIONS, NOV 2023

MATHEMATICAL FOUNDATIONS FOR DATA SCIENCE-III [MAT 3151]

Marks: 50 **Duration: 180 mins.**

A

Answer all the questions.

Instructions to Candidates:
Missing data may be suitably assumed

1) If G is a self complementary graph on n vertices, then prove that n is of the form 4k or 4k+1, k>=0.
Give an example for a regular self complementary graph. (3)

A)

B) Prove that if $n \geq 6,$ either G or $\overline{G}$ contains a triangle. Draw a graph on 8 vertices that contains no induced subgraph which is either K_3 or 4 isolated vertices. (3)

C) Let G be a connected graph. Prove that G is Eulerian if and only if all the vertices are of even degree. (4)

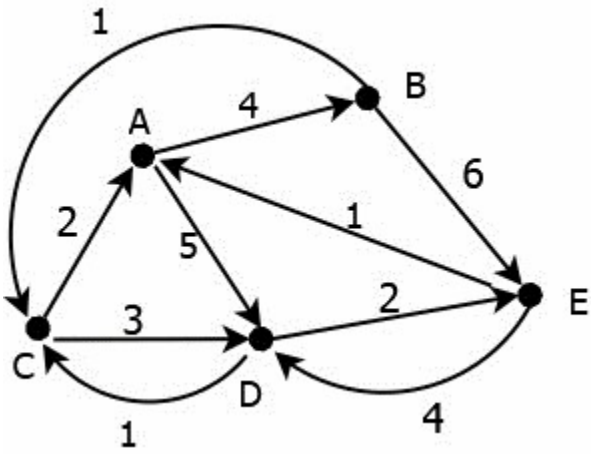
2) Prove that in a tree on n vertices, the centre of tree is either a single vertex or a pair of adjacent vertices. Draw a tree with 8 vertices and diameter 2. (3)

A)

B) Define closeness centrality. Obtain the closeness centrality of all the vertices in (i) a cycle graph on n vertices and (ii) a complete bipartite graph, $K_{p.q}$. (3)

C) Using Floyd Warshall algorithm, obtain all pairs shortest paths from every vertex to every other vertices for the graph as shown below.

(4)



3) Obtain the determinant of adjacency matrix of (i) a cycle graph on n vertices (ii) a path graph on n vertices (iii) a complete graph on 4 vertices by obtaining all the elementary spanning subgraphs and using the formula

A) $\det(A(G)) = \sum(-1)^{n-c_1(H)-c(H)}\ 2^{c(H)}$, where summation runs over all elementary spanning subgraphs H of G and $c_1(H)$ and c(H) are the number of components which are K_2's and cycles respectively. (3)

B) Find the smallest number n with $\tau(n) = 15$, where $\tau(n)$ denotes the number of divisors of n. (3)

C) Encipher the message "HAVE A NICE TRIP" using a Vigenère cipher with the keyword "MATH". (4)

4) Prove that $\tau(n)$, the number of divisors of n is odd if and only if n is a perfect square. (3)

A)

B) Find the number of integers in the set S={1,2,3,...,6300} that are divisible either by 3 or 4. Also, find the number of integers that are divisible by neither 3 nor 4. (3)

C) i) Write binary quadratic form with discriminant d = 9.

ii) Show that, the binary quadratic forms $f(x,y)=x^2+y^2$ and $g(x,y)=x^2-2xy+2y^2$ are equivalent. (4)

5) Prove that, 7 divides $111^{333}+333^{111}$. (3)

A)

B) Let $m, n \in \mathbb{Z}$ such that gcd(m,n)=1. Show that, (3)

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \ (mod\, mn).$$

C) i) Find the number of zeros at the end of the 50!.

ii) Compute the day for the date August 15, 2017, using the formula
$d \equiv N + [2.6M - 0.2] + Y + [Y/4] + [C/4] - 2C - (1 + L)[M/11](mod\ 7)$ (4)

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