Exam Date & Time: 09-Jan-2024 (02:30 PM - 05:30 PM)

# MANIPAL ACADEMY OF HIGHER EDUCATION

**FINITE ELEMENT METHODS [MME 3152]**

**Marks: 50** **Duration: 180 mins.**

**Des**

**Answer all the questions.** Section Duration: 180 mins

1A) Solve the following system of equations using the Gaussian elimination method: (4)

$$2x_1 + 2x_2 + 1x_3 = 9$$
$$1x_1 + 1x_2 + 1x_3 = 6$$
$$2x_1 + 1x_2 = 4$$

1B) Evaluate the following integral by using the two-point Gauss integration. (3)

$$\int_{-1}^{1}\int_{-1}^{1} x^2 y^2 \; dxdy$$

Useful formulae:

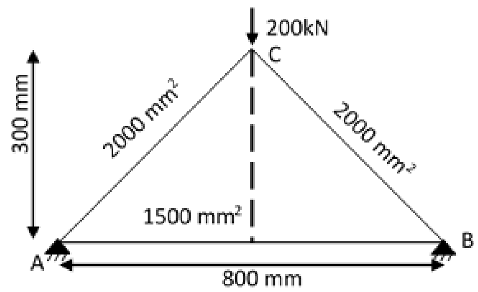
Location of Gauss points:

$r_1, r_2 = \pm 0.577$

Weights = 1

1C) Briefly illustrate the advantages and disadvantages of FEM. (3)

2A) Determine the nodal displacements for the three-bar truss shown in the figure. Find the support reactions also. Take modulus of elasticity as 200 GPa.




Useful formula: (4)

$$[k]_e = \frac{E_e\,A_e}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

2B) Using penalty approach to handle the boundary conditions, find (a) displacement at each node, (b) stresses and strains for each element. (3)

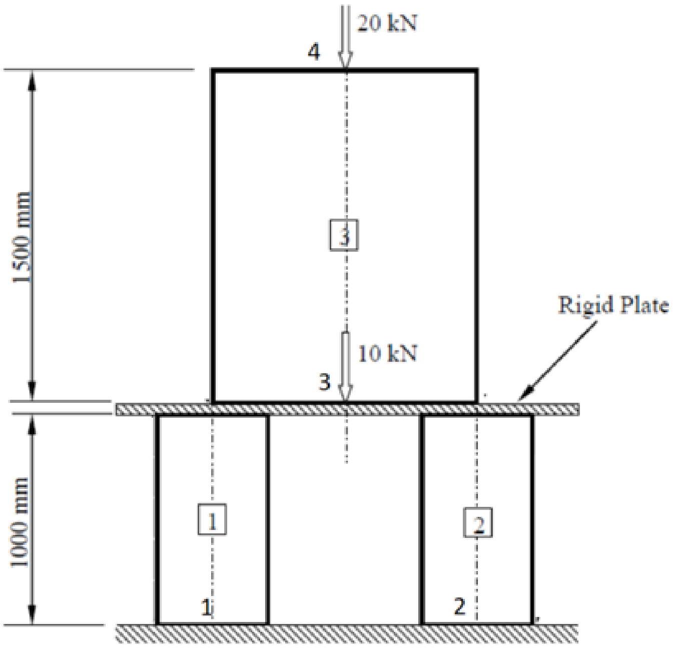
Here A1=100X$10^3$ $mm^2$, E1=90 MPa, A2=100X$10^3$ $mm^2$,

E2=100 MPa, A3=250X$10^3$ $mm^2$, E3=120 MPa

The global stiffness matrix K is provided.

$$[K]=10^3 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 9 & 0 & -9 & 0 \\ 0 & 10 & -10 & 0 \\ -9 & -10 & (9+10+20) & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

(While calculating the global stiffness matrix above, the dimensions were expressed in mm, and the unit used for force is kN.)




2C) Using the principle of minimum potential energy (energy approach), find the element stiffness matrix for a two-noded bar element with area A, elastic modulus E, and length L. (3)

Useful formulae:

For 1D problem, the potential energy for each 1D element $\Pi^e$ can be expressed as

$$\Pi^e = \frac{1}{2}\int_{l^{(e)}} \{\sigma\}^T\{\varepsilon\}A dx - \int_{l^{(e)}} \{u\}^T\{b\}A dx - \int_{l^{(e)}} \{u\}^T\{T\}dx - \sum_k u_k P_k$$

where, {σ} and {ε} are the axial stress and axial strain,
{b} is the body force, N/unit length
{T} is the traction force, N/unit length
{u} is the axial displacement, m
$P_i$ is the point load action at point i, N
$u_i$ is the axial displacement at point i, m

$\varepsilon = \{B\}\{q\}$ & $\sigma = E^{(e)}\{B\}\{q\}$ $\{\sigma\}^T = \{q\}^T\{B\}^T E^{(e)}$

Converting from x coordinate to $\zeta$ coordinate, and changing the limits from $[0$ to $l_e]$ to $[\zeta = -1$ to $\zeta = +1]$. We have

$$\frac{d\zeta}{dx} = \frac{2}{l^{(e)}}$$

$$\therefore dx = \frac{l^{(e)}}{2} d\zeta$$

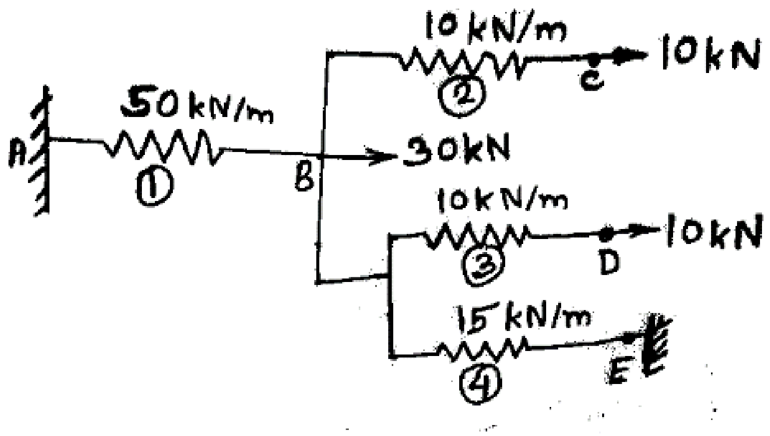
3A) For the spring system illustrated in the figure, form the {F} = [K] {Q} relationship and find

(a) the global stiffness matrix,

(b) the displacements of each node

(c) reaction at supports.

Points A and E are fixed.




(3)

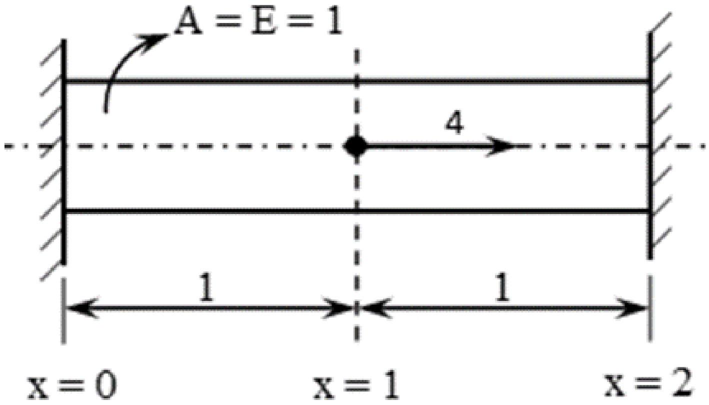
Useful formula:

Element characteristic matrix:

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

3B) Illustrate h and p methods of convergence. (3)

3C) A rod fixed at its ends is subjected to a point load of 4 units in the axial direction as shown in the figure. Use the Rayleigh-Ritz method to find a quadratic polynomial solution of the displacement function, $u=a_0+a_1X+a_2X^2$ . The bar has the following details: Length L=2 units; Area of cross section A=1 unit; Elastic modulus E=1 unit; Point load p=4 units in X-direction. (4)



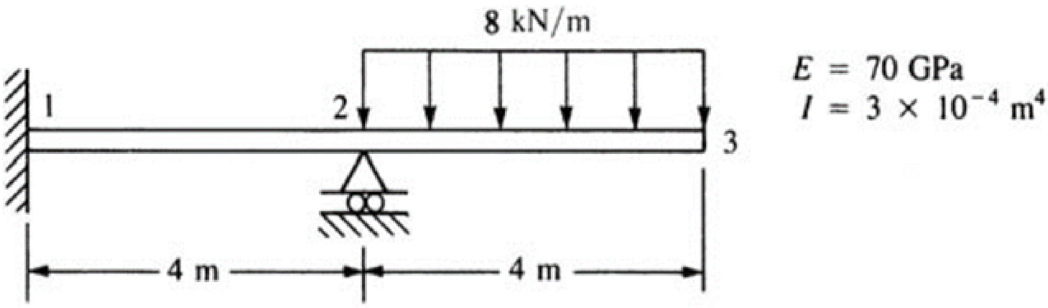

Useful formulae:

The equation of axial strain for 1D case is $\varepsilon = \frac{du}{dx}$

The potential energy of a system has the following form

$\Pi = \frac{1}{2}\int_0^L \sigma\varepsilon A dx - \int_0^L ubA dx - \int_0^L uT dx - \sum_i u_i P_i$

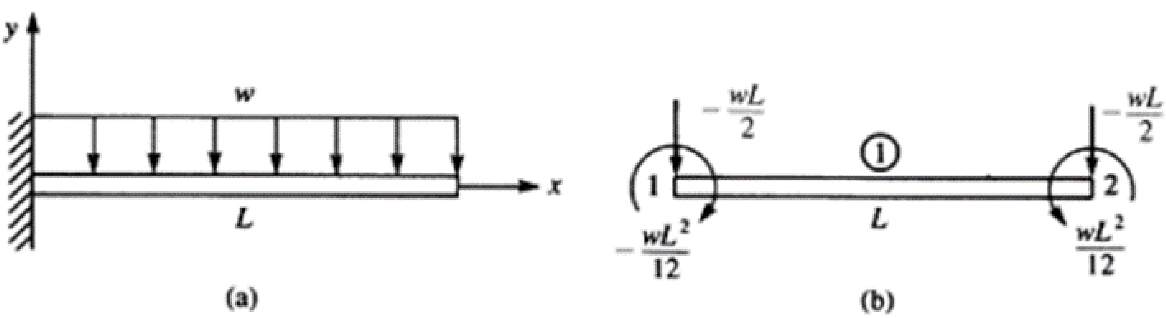
4A) Evaluate the nodal displacements and slopes for the beam shown in the figure. (4)



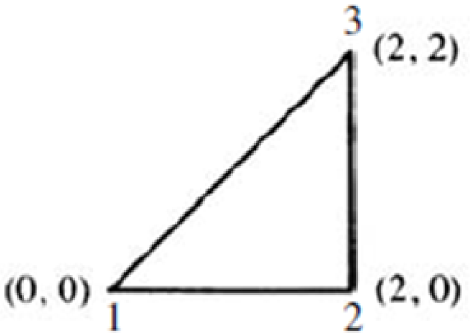

Useful Formulae:

$$[k_e] = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

For the conversion of given distributed load into equivalent nodal forces and moments, the following may be used:



4B) For the axisymmetric element shown in the figure, evaluate the stiffness matrix. Take Young's Modulus = $30\times10^6$ psi and Poison's ratio = 0.25. Coordinates of vertices are in inches. (4)




<u>Useful formulae:</u>

The element stiffness matrix is:

$$\boxed{\therefore [k]^e = 2\pi r' A_e [B']^T [D][B']}$$

$$where, A_e = \frac{1}{2} Det[J]$$

Jacobian matrix is given by:

$$[J] = \begin{bmatrix} r_{13} & z_{13} \\ r_{23} & z_{23} \end{bmatrix}$$

Strain displacement matrix B' calculated at the centroid is given by:

$$\frac{1}{Det[J]}\begin{bmatrix} z_{23} & 0 & z_{31} & 0 & z_{12} & 0 \\ 0 & r_{32} & 0 & r_{13} & 0 & r_{21} \\ r_{32} & z_{23} & r_{13} & z_{31} & r_{21} & z_{12} \\ Det[J]\frac{N_1}{r'} & 0 & Det[J]\frac{N_2}{r'} & 0 & Det[J]\frac{N_3}{r'} & 0 \end{bmatrix}$$

$$N_1 = N_2 = N_3 = 1/3 \qquad \& \qquad r' = \frac{r_1 + r_2 + r_3}{3}$$

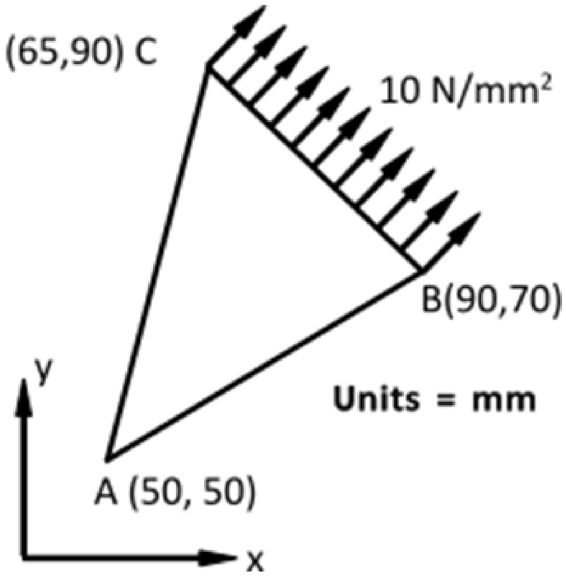
where $r^1$ is the radius of the centroid.

D matrix is given by:

$$\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 & \nu \\ \nu & 1-\nu & 0 & \nu \\ 0 & 0 & \frac{1-2\nu}{2} & 0 \\ \nu & \nu & 0 & 1-\nu \end{bmatrix}$$

4C) Illustrate why beam element formulation is called sub-parametric formulation? (2)

5A) Calculate the equivalent nodal force components for a 3 noded CST element due to a traction force of 10 N/mm$^2$ acting normal to face BC and due to self-weight of the element. Assume, density of the element as 8000 kg/m$^3$ . The thickness of the element is 5 mm, and nodal coordinates of the element are shown in the figure.



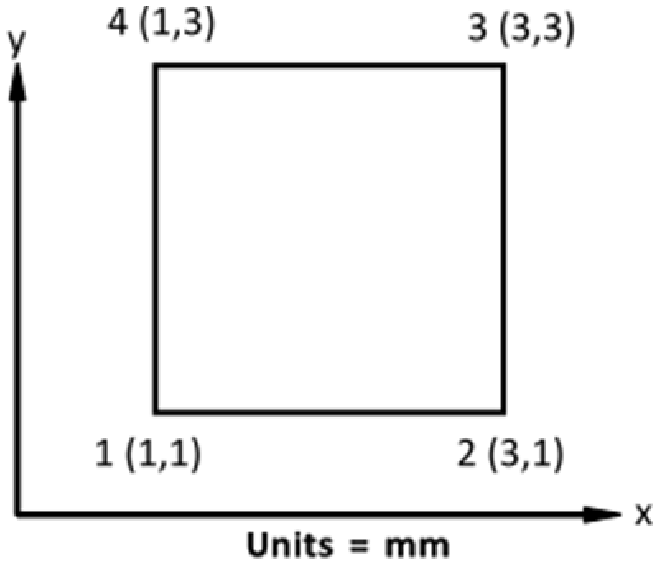

Useful formulae: (4)

$$\{b\}^e = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix}^e = \left(\frac{A_e t_e}{3}\right)\begin{Bmatrix} b_x \\ b_y \\ b_x \\ b_y \\ b_x \\ b_y \end{Bmatrix}$$

$$\{T\}^e = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix}^e = \left(\frac{l_{side} t_e}{2}\right)\begin{Bmatrix} T_x \\ T_y \\ T_x \\ T_y \\ T_x \\ T_y \end{Bmatrix}$$

$$A = \frac{1}{2}\begin{vmatrix} 1 & x_A & y_A \\ 1 & x_B & y_B \\ 1 & x_C & y_C \end{vmatrix}$$

5B) For the quadrilateral element shown in the figure, assume plane stress conditions with E = 210 GPa, Poisson's Ratio = 0.3. Evaluate (i) elasticity matrix, (ii) determinant of the Jacobian matrix at the centroid of the element. (4)




Useful formulae:

$$[D]=\frac{E}{1-\nu^2}\begin{bmatrix}1 & \nu & 0\\ \nu & 1 & 0\\ 0 & 0 & \frac{1-\nu}{2}\end{bmatrix}$$

$$[J]=\begin{bmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{bmatrix}=\begin{bmatrix}\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta}\\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}\end{bmatrix}$$

$$N_1=\frac{(1-\zeta)(1-\eta)}{4} \qquad N_2=\frac{(1+\zeta)(1-\eta)}{4}$$

$$N_3=\frac{(1+\zeta)(1+\eta)}{4} \qquad N_4=\frac{(1-\zeta)(1+\eta)}{4}$$

$$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4$$
$$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$$

5C) What are shape functions in finite element method and draw a schematic of variation of shape functions for a linear beam element. (2)

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