Exam Date & Time: 04-Dec-2023 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

FINITE ELEMENT METHODS

FINITE ELEMENT METHODS [MME 3152]

Marks: 50 **Duration: 180 mins.**

END EXAM

Answer all the questions. Section Duration: 180 mins

Assume missing data/information (if any)
Draw neat sketches wherever necessary

1A) Solve the following system of equations using the Gaussian elimination method: (4)

$x_1 - x_2 + 3x_3 = 10$

$2x_1 + 3x_2 + x_3 = 15$

$4x_1 + 2x_2 - x_3 = 6$

1B) Evaluate the following integral by using the two-point Gauss integration. (3)

$$I = \int_{-1}^{1}\int_{-1}^{1}(r^2 s^3 + r s^4)\,dr\,ds$$

Useful formulae:

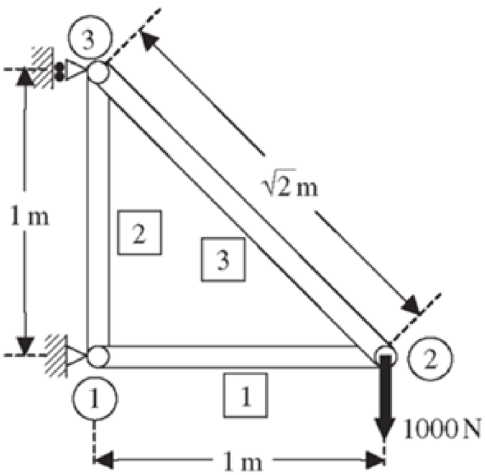
Location of Gauss points:

$r_1, r_2 = \pm 0.577$

Weights = 1

1C) Briefly illustrate the applications of FEM. (3)

2A) Consider the plane truss structure shown in the figure. The structure is made of three planar truss members as shown, and a vertical downward force of 1000 N is applied at node 2. The figure also shows the numbering of the elements used (labelled in squares), as well as the numbering of the nodes (labelled in circles). Evaluate the nodal displacements. Here Young's modulus E =70 GN/m^2, Area A=0.1 m^2. Node 3 has roller support (allows vertical movement only) and node 1 has fixed support. (4)

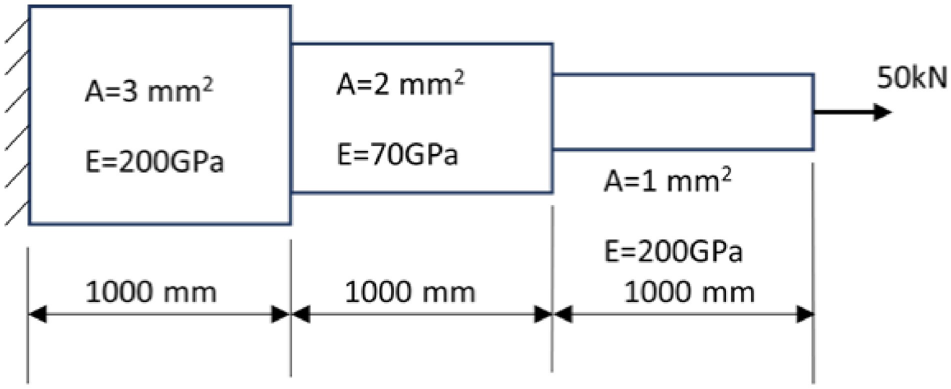


Useful formula:

$$[k]_e = \frac{E_e A_e}{l_e}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

2B) Using penalty approach to handle the boundary condition, find (a) displacement at each node, (b) stresses and strains for each element.

The global stiffness matrix K is provided.



(3)

$$K = \begin{bmatrix} 600 & -600 & 0 & 0 \\ -600 & 740 & -140 & 0 \\ 0 & -140 & 340 & -200 \\ 0 & 0 & -200 & 200 \end{bmatrix}$$

.

(While calculating the global stiffness matrix above, the dimensions were expressed in mm, and the unit used for force is N.)

2C) Using the principle of minimum potential energy (energy approach), find the body force vector for a two-noded bar element with area A, elastic modulus E, and length L. (3)

Useful formulae:

$$u = \{N\}\{q\}$$

$$\therefore \{u\}^T = \{q\}^T\{N\}^T$$

We have,

$$N_1 = \frac{1-\zeta}{2} \qquad \& \qquad N_2 = \frac{1+\zeta}{2}$$

$$Also,\ \int_{-1}^{+1} N_1 d\zeta = \int_{-1}^{+1} N_2 d\zeta = 1$$

For 1D problem, the potential energy for each 1D element Π^e can be expressed as

$$\Pi^e = \frac{1}{2}\int_{l^{(e)}} \{\sigma\}^T\{\varepsilon\}A dx - \int_{l^{(e)}} \{u\}^T\{b\}A dx - \int_{l^{(e)}} \{u\}^T\{T\}dx - \sum_k u_k P_k$$

where, $\{\sigma\}$ and $\{\varepsilon\}$ are the axial stress and axial strain,
$\{b\}$ is the body force, N/unit length
$\{T\}$ is the traction force, N/unit length
$\{u\}$ is the axial displacement, m
P_i is the point load action at point i, N
u_i is the axial displacement at point i, m

$$\frac{d\zeta}{dx} = \frac{2}{l^{(e)}}$$

$$\therefore dx = \frac{l^{(e)}}{2} d\zeta$$

$$[\zeta = -1 \text{ to } \zeta = +1]$$

3A) For the spring system shown in the figure, find

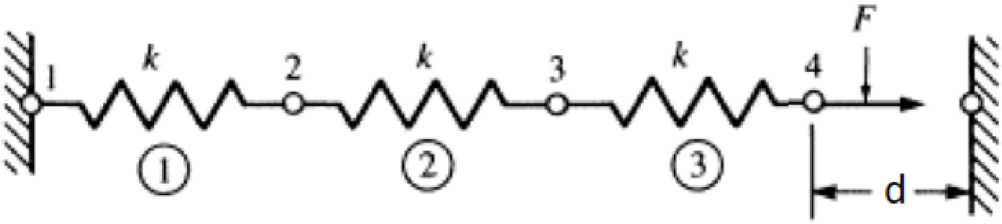
(a) the global stiffness matrix,

(b) the displacements of nodes 2 and 3,

(c) the local element forces.

Node 1 is fixed while node 4 is given a fixed known displacement d = 10 mm.

The spring constants are all equal to k =100 kN/m. (3)



Useful formula:

Element characteristic matrix:

$$\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

3B) Using the "Useful formulae" given below, and with the help of a neat labelled diagram, derive the shape functions for 9 noded quadrilateral element. (3)

Useful formulae:

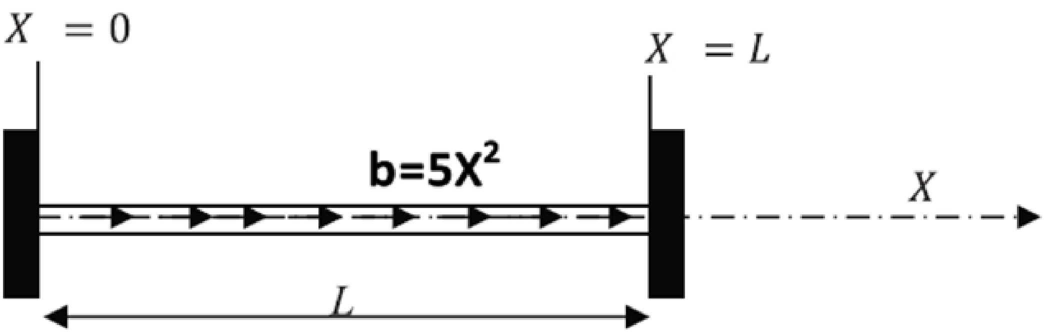
Shape functions for the 1D horizontal element are given by:

$$N_{1\zeta} = -\frac{\zeta(1-\zeta)}{2}$$

$$N_{2\zeta} = \frac{\zeta(1+\zeta)}{2}$$

$$N_{3\zeta} = 1-\zeta^2$$

3C) A steel rod fixed at its ends is subjected to a **varying** body force b, as shown in the figure. Use the Rayleigh-Ritz method to find a quadratic polynomial solution of the displacement function, $u=a_0+a_1X+a_2X^2$. The bar has the following details: Length L=2 m; Area of cross section A=250 x 250 mm^2; Elastic modulus E=1x10^5 Pa; **Varying** body force, **b=5X^2** N/m in X-direction. (4)



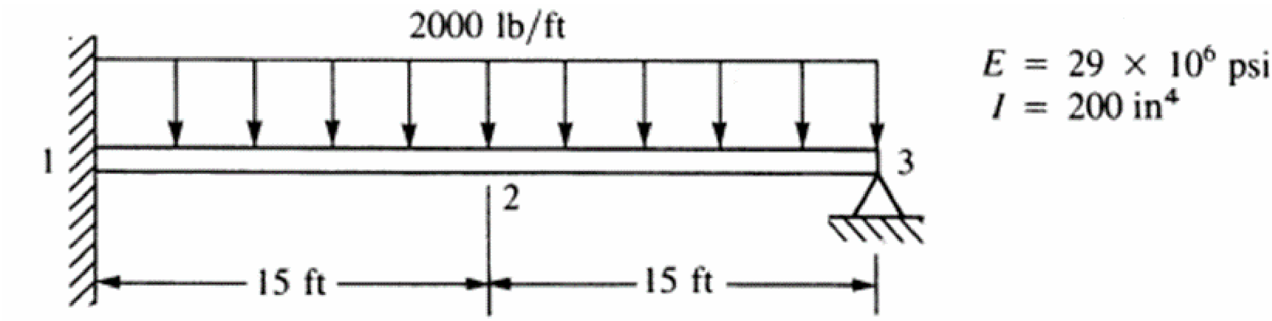
Useful formulae:

The equation of axial strain for 1D case is $\varepsilon = \frac{du}{dx}$

The potential energy of a system has the following form

$$\Pi = \frac{1}{2}\int_0^L \sigma\varepsilon A dx - \int_0^L ubA dx - \int_0^L uT dx - \sum_i u_i P_i$$

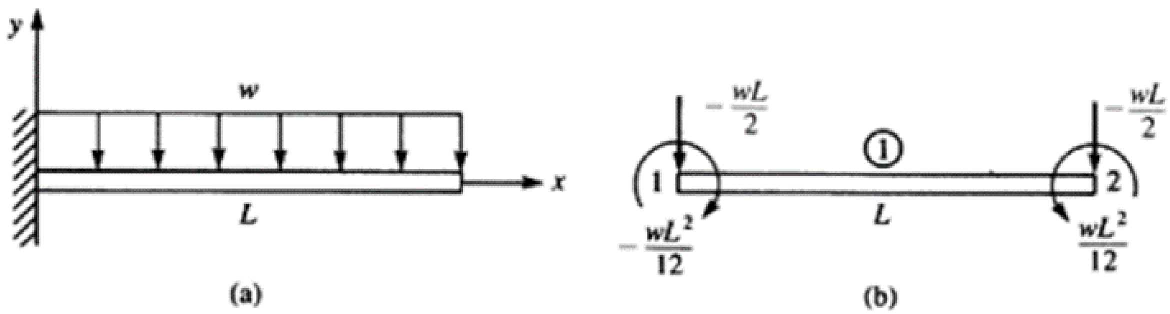
4A) Evaluate the nodal displacements and slopes for the beam shown in the figure. (4)



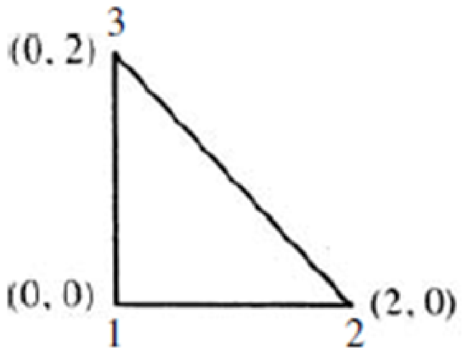
Useful Formulae:

$$[k_e] = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

For the conversion of given distributed load into equivalent nodal forces and moments, the following may be used:



4B) For the axisymmetric element shown in the figure, evaluate the stiffness matrix. Take Young's Modulus = 30×10^6 psi and Poison's ratio = 0.25. Coordinates of vertices are in inches. (4)



Useful formulae:

The element stiffness matrix is:

$$\boxed{\therefore [k]^e = 2\pi r' A_e [B']^T [D][B']}$$

$$where, A_e = \frac{1}{2} Det[J]$$

Jacobian matrix is given by:

$$[J]=\begin{bmatrix} r_{13} & z_{13} \\ r_{23} & z_{23} \end{bmatrix}$$

Strain displacement matrix B' calculated at the centroid is given by:

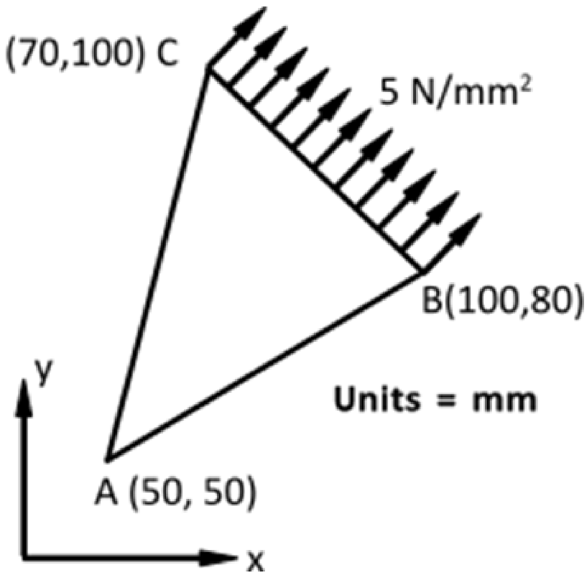
$$\frac{1}{Det[J]}\begin{bmatrix} z_{23} & 0 & z_{31} & 0 & z_{12} & 0 \\ 0 & r_{32} & 0 & r_{13} & 0 & r_{21} \\ r_{32} & z_{23} & r_{13} & z_{31} & r_{21} & z_{12} \\ Det[J]\frac{N_1}{r'} & 0 & Det[J]\frac{N_2}{r'} & 0 & Det[J]\frac{N_3}{r'} & 0 \end{bmatrix}$$

$$N_1 = N_2 = N_3 = \tfrac{1}{3} \qquad \& \qquad r' = \frac{r_1 + r_2 + r_3}{3}$$

where r^1 is the radius of the centroid.

4C) A bicycle frame is subjected to both axial loads and planar bending loads. Suggest element type to be used to model this in FEM. Write a neat diagram showing the local and global displacements and rotations of element. (2)

5A) Calculate the equivalent nodal force components for a 3 noded CST element due to a traction force of 5 N/mm^2 acting normal to face BC and due to self-weight of the element. Assume, density of the element as 7800 kg/m^3 . The thickness of the element is 10 mm, and nodal coordinates of the element are shown in the figure. (4)



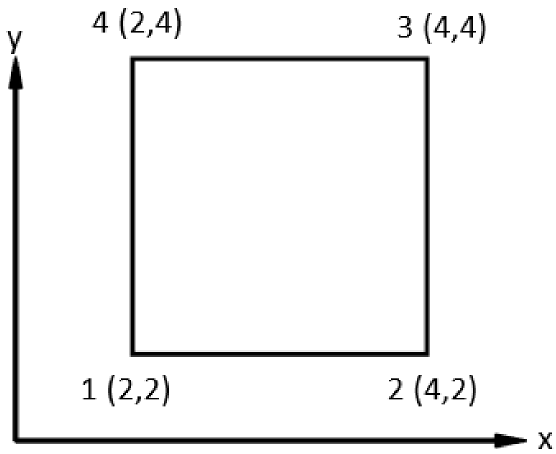
Useful formulae:

$$\{b\}^e = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{Bmatrix}^e = \left(\frac{A_e t_e}{3}\right)\begin{Bmatrix} b_x \\ b_y \\ b_x \\ b_y \\ b_x \\ b_y \end{Bmatrix}$$

$$\{T\}^e = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{Bmatrix}^e = \left(\frac{l_{side}\, t_e}{2}\right) \begin{Bmatrix} T_x \\ T_y \\ T_x \\ T_y \\ T_x \\ T_y \end{Bmatrix}$$

5B) What is Jacobian in finite element analysis and explain its significance?

Also, calculate the Jacobian matrix at the centroid of the quadrilateral element shown in the figure by using 1x1 point integration rule.



(4)

<u>Useful formulae:</u>

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix}$$

$$N_1 = \frac{(1-\zeta)(1-\eta)}{4} \qquad N_2 = \frac{(1+\zeta)(1-\eta)}{4}$$

$$N_3 = \frac{(1+\zeta)(1+\eta)}{4} \qquad N_4 = \frac{(1-\zeta)(1+\eta)}{4}$$

$x = N_1x_1 + N_2x_2 + N_3x_3 + N_4x_4$
$y = N_1y_1 + N_2y_2 + N_3y_3 + N_4y_4$

5C) Draw a linear hexahedral element and mention the degrees of freedom at every node on the element. How many shape functions are required for the element? (2)

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