MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

VII SEMESTER B.TECH

MAKE-UP EXAMINATIONS, JANUARY 2023

SUBJECT: GRAPHS AND MATRICES [MAT 4054] (PE)

Instructions to Candidates:

- Answer ALL the questions.

Date: **05-01-2023** Time: **02.30 PM – 05.30 PM** Max. Marks: **50**

1A. For every graph G, show that either G or $\bar{G}$, is connected **(3 Marks)**

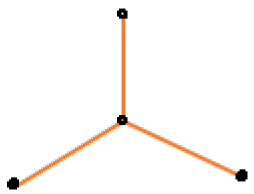
1B. Show that a self-complementary graph has 4n or 4n + 1 points **(3 Marks)**

1C. Show that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ lines. Hence deduce that, if q > $\frac{1}{2}(p-1)(p-2)$, then G must be connected **(4 Marks)**

2A. Let G be (p, q) graph. Show that G is tree if and only if every two points of G are joined by a unique path **(3 Marks)**

2B. For any plane map with p vertices, q edges and r regions, show that p – q + r = 2 **(3 Marks)**

2C. Obtain the chromatic polynomial of the graph given below.

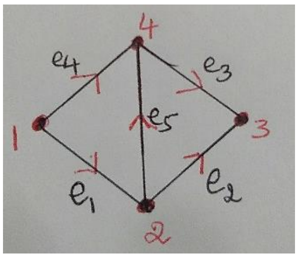


Use the recurrence relation f(G, t) = f(G + e, t) + f(G.e, t) **(4 Marks)**

3A. Show that if a graph G is Eulerian then G is connected and every point of G is of even degree **(3 Marks)**

3B. For a non-trivial simple (p, q) graph G, show that $\frac{p}{\beta_0} \leq \chi(G) \leq p - \beta_0 + 1$, β_0 denotes point independence number and $\chi(G)$ denotes the chromatic number **(3 Marks)**

3C. Consider the graph given below.



By considering the spanning tree formed by $\{e_1, e_2, e_3\}$, find the Moore Penrose inverse of the incidence matrix **(5 Marks)**

4A. If G is a connected graph on n vertices, show that rank of the {0, 1, -1} incidence matrix Q(G) has rank n-1 **(2 Marks)**

4B. Let G be a bipartite graph with adjacency matrix A. If λ is an eigenvalue of A with multiplicity k, then show that $-\lambda$ is also an eigenvalue of A with multiplicity k **(3 Marks)**

4C. Let G be a graph with n vertices, m edges and let λ_1 be the largest eigen value of G. Then show that $\lambda_1 \leq \sqrt{\frac{2m(n-1)}{n}}$ **(4 Marks)**

5A. Show that eigenvalues of cycle C_n are $2\cos\left(\frac{2k\pi}{n}\right)$, $n = 1, 2, \ldots, n$ **(3 Marks)**

5B. Let L denote the Laplacian matrix of a simple graph G. Show that L is a symmetric and positive definite matrix **(3 Marks)**

5C. Let G be a simple graph with at least one edge. Let λ_1 be the largest eigenvaluee of the Lapacaian matrix of G. Show that $\lambda_1 \geq \Delta(G) + 1$, where λ_1 denotes the largest eigen value of the Laplacian matrix and $\Delta(G)$ denotes the maximum degree of G **(4 Marks)**