Reg. No.									

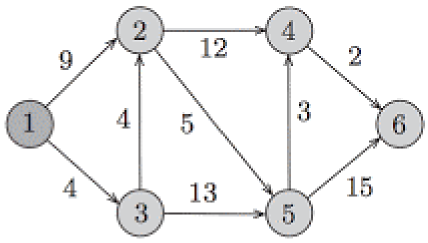
MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

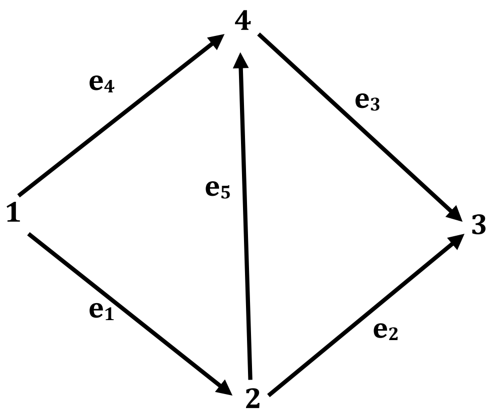
SEVENTH SEMESTER
END SEMESTER EXAMINATIONS, DECEMBER 2023

Graphs & Matrices [MAT 4054]
REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 2 December 2023** **Max. Marks: 50**

Instructions to Candidates:
- Answer **ALL** the questions.

Q.NO	Questions	Marks	CO	BTL
1A.	For any graph G on 6 points, show that G or the complement of G contains a triangle	3	1	4
1B.	Show that a simple graph with p vertices and k components cannot have more than (p-k)(p-k+1)/2 edges. Hence show that if q > (p-1)(p-2)/2 in a graph G, then G must be connected	3	1	4
1C.	Use Dijkstra's algorithm to find shortest paths from 1 to all other vertices of the graph 	4	2	3
2A.	Let G be a connected graph and v be a cut vertex of G. Then show that v is not a cut vertex of complement of G	3	1	4
2B.	Show that a graph is bipartite if and only if all its cycles are of even length	3	1	4
2C.	Let M be the {0, 1} incidence matrix of a simple graph G. Show that M is totally unimodular if and only if G is bipartite.	4	5	4
3A.	Show that a simple (p, q) graph G is a tree if and only it is acyclic and p = q + 1	3	2	4
3B.	For any graph G, prove that $k(G) \leq \lambda(G) \leq \delta(G)$, *where* $k(G), \lambda(G)$ *and* $\delta(G)$ respectively denote vertex connectivity, edge connectivity and minimum degree of the graph G	3	3	4
3C.	If G is a connected graph on n vertices, let Q(G) denote the {0, 1, -1} incidence matrix and L(G) denote the Laplacian matrix of G.	4	5	4

	Show that rank of Q(G) = n-1. Hence deduce that rank of Q(G) = n-k and rank of L(G) = n-k, if G is disconnected and has k components			
4A.	Suppose that University examinations are to be scheduled. Suppose that there are 7 courses numbered 1 through 7. Suppose that the following pairs of courses have common students: 1 & 2, 1 & 3, 1 & 4, 1 & 7, 2 & 3, 2 & 4, 2 & 5, 2 & 7, 3 & 4, 3 & 6, 3 & 7, 4 & 5, 4 & 6, 5 & 6, 5 & 7, 6 & 7. Schedule a timetable that requires a minimum number of days to conduct an examination using graph colouring	3	3	3
4B.	For a plane map with p vertices, q edges and f faces show that p + q – f = 2. Hence show that if G is maximal planar graph then every face is a cycle and that q = 3p -6	3	3	4
4C.	For n ≥ 2, show that the eigen values of the cycle C_n are $2\cos(2\pi k/n)$, k = 1, 2, …,n. Hence obtain eigen values of the Laplacian matrix of C_n.	4	4&5	4
5A.	Let G be a graph with n vertices, m edges and let $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$ be the eigen values of G. Then show that $\lambda_1 \leq \sqrt{\frac{2m(n-1)}{n}}$	3	4	4
5B.	Let G be a bipartite graph with adjacency matrix A. If λ is an eigen value A with multiplicity k, then show that -λ is also an eigen value of A with the same multiplicity	3	4	4
5C.	Consider the following graph:  By taking the spanning tree formed by {e_1, e_2, e_3}, obtain the Moore-Penrose inverse of the incidence matrix of the graph.	4	5	3