Question Paper

Exam Date & Time: 28-Nov-2023 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATIONS NOVEMBER/DECEMBER 2023
I SEMESTER BSc(APPLIED SCIENCES) IN ENGG.

MATHEMATICS - 1 [IMA 111]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

Missing data, if any, may be suitably assumed

1) A) Verify Cauchy's mean value theorem for the functions $\sin x$ and $\cos x$ in the interval $(\frac{\pi}{6}, \frac{\pi}{3})$. (3)

B) If $u = \log_e\left(\frac{x^4+y^4}{x+y}\right)$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$. (3)

C) Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{\frac{1}{x}}$ (4)

2) A) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}, y = e^{2t}\cos 3t, z = e^{2t}\sin 3t$. Find $\frac{du}{dt}$ as the total derivative and verify the result by direct substitution. (3)

B) Find $\frac{ds}{d\theta}$ for the curve $r = a(1-\cos\theta)$. (3)

C) Expand $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ upto second degree terms. (4)

3) A) Show that the tangents drawn at the extremities of any chord of the cardiode $r = a(1+\cos\theta)$ which passes through the pole are perpendicular to each other. (3)

B) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$. Prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + 2n^2y_n = 0$. (3)

C) Find the maximum and minimum value of the function $x^3 + y^3 - 3axy$. (4)

4) Find the radius of curvature for the cycloid $x = a(t+\sin t), y = a(1-\cos t)$. (3)

A)

B) Test for the convergence or divergence of the series $\sum_{n=1}^{\infty} ne^{-n^2}$. (3)

C) Test for the convergence or divergence of the series $3x + 3^4x^4 + 3^9x^9 + \cdots . 3^{n^2}x^{n^2} + \cdots \ldots$ (4)

5) Using reduction formula, Evaluate $\int_{x=0}^{a} x\sqrt{ax - x^2}dx$ (3)

A)

B) Find the area of the loop of the curve $ay^2 = x^2(a - x)$ (3)

C) Show that evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$ (4)

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