Question Paper

Exam Date & Time: 13-Jan-2024 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATIONS NOVEMBER/DECEMBER 2023
I SEMESTER BSc(APPLIED SCIENCES) IN ENGG.
MATHEMATICS-I [IMA 111 - S2]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

Missing data, if any, may be suitably assumed

1) (3)

A) Verify Cauchy's mean value theorem for the functions e^x and e^{-x} in the interval (a, b).

B) (3)

Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \log u$ where $\log u = \frac{x^3+y^3}{3x+4y}$.

C) (4)

Find the value of a and b in order that $\lim_{x\to 0} \frac{x(1+a\cos x)-b\sin x}{x^3}$ is equal to 1.

2) (3)

If $u = x \log xy$, where $x^3 + y^3 + 3xy = 1$. Find $\frac{du}{dx}$.

A)

B) (3)

Find $\frac{ds}{d\theta}$ for the curves $x = a(1 - \cos\theta), y = a(\theta + \sin\theta)$.

C) (4)

Expand $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x - 1)$ and $(y - 1)$ upto second degree terms.

3) (3)

A) Find the angle of intersection of the cardiode $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.

B) (3)

If $y = e^{a\sin^{-1}x}$. Prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

C) (4)

Find the maximum and minimum value of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.

4) (3)

A) Find the radius of curvature at the point $(\frac{3a}{2}, \frac{3a}{2})$ of the equation $x^3 + y^3 = 3axy$.

B) Test for the convergence or divergence of the series $\sum_{n=0}^{\infty} \frac{2n^3+5}{4n^5+1}$. (3)

C) Test for the convergence or divergence of the series $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + - - - - -$ (4)

5) Using reduction formula evaluate $\int_0^1 \frac{x^9}{\sqrt{1-x^2}} dx$. (3)

A)

B) Find the area enclosed by the curve $a^2x^2 = y^3(2a - y)$. (3)

C) Find the equation of the circle of curvature of $2xy + x + y = 4$ at the point (1, 1). (4)

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