Question Paper

Exam Date & Time: 13-Jan-2024 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

INTERNATIONAL CENTRE FOR APPLIED SCIENCES
END SEMESTER THEORY EXAMINATION - NOVEMBER/DECEMBER 2023
III SEMESTER B.Sc. (APPLIED SCIENCES) IN ENGG.

Mathematics -III [IMA 231 - S2]

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

Missing data, if any, may be suitably assumed

1) (4)

A) A businessman goes to the hotel X, Y and Z respectively 20%, 50% and 30% of the time. It is known that 5%, 4%, 8% of the rooms in X, Y, Z have faulty plumbing. What is the probability that his room having faulty plumbing is assigned to hotel X, Y and Z.

B) (3)

Derive mean and variance of Chi-square distribution.

C) (3)

How many r-digits ternary sequences are there with
(i) Even number of 0.
(ii) Even number of 0 and even number of 1.
(iii) At least one 0 and at least one 1.

2) (4)

A) If X, Y and Z are uncorrelated random variables with standard deviation 5, 12 and 9 respectively. If U = X + Y and V = Y + Z, evaluate the correlation coefficient between U and V.

B) (3)

An alloy contains certain percentage of lead say X, which may be considered as a pdf

$$f(x) = \begin{cases} \frac{3}{5}[10^{-5}x(100-x)]; & 0 \le x \le 100 \\ 0; & elsewhere \end{cases}.$$

C) (3)

Find the number of ways that can be obtained change for Rs. 10/- in terms of Rs. 5/-, Rs. 2/- and Rs. 1/-.

3) (4)

A)

Solve the LPP by Simplex method. Max Z = $4x_1 + 10x_2$
Subject to the constraints,
$2x_1 + 2x_2 \le 50$;

$2x_1 + 5x_2 \leq 100$;
$2x_1 + 3x_2 \leq 90$;
And $x_1 \geq 0$; $x_2 \geq 0$.

B) The height of 1000 soldiers are found to have normal distribution. Of them 564 are found to be within 3cm of the mean height of 170cm. Find the standard deviation of X. (3)

C) The joint probability density function of the random variable (X, Y) is given by (3)

$f(x,y) = kxye^{-(2x^2+y^2)}, 0 < x, y < \infty$.

(i) Find k.

(ii) Show that X and Y are independent random variables.

4) Find the moment generating function of Exponential distribution and then derive its mean and variance. (4)

A)

B) A manufacturer produces two types of models A and model B. Model A requires 4 hours of grinding and 2 hours of polishing whereas, Model B requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours per week and each polisher works for 60 hours per week. Profit on model A is Rs. 3 and model B is Rs. 4. Formulate an LPP to make a maximum profit in a week. (3)

C) Find pdf of $Y = -2logX$, where $f(x) = \begin{cases} 1; & 0 \leq x \leq 1 \\ 0; & elsewhere \end{cases}$. (3)

5) (4)

A) If X_1 and X_2 are independent random variables and have standard normal distribution. Find the pdf of $\frac{X_1}{X_2}$.

B) Solve the given LPP by graphical method. (3)

Max Z $= 10x_1 + 20x_2$

Subject to the constraints,

$3x_1 + 4x_2 \leq 30$;

$5x_1 + 2x_2 \geq 20$

and $x_1 \geq 0$; $x_2 \geq 0$.

C) If X has pdf $f(x) = \lambda e^{-\lambda(x-2)}$ $if\ x \geq 2$. Find moment generating function and hence find mean and variance of X. (3)

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