# Question Paper

Exam Date & Time: 27-Nov-2023 (09:30 AM - 12:30 PM)

## MANIPAL ACADEMY OF HIGHER EDUCATION

**INTERNATIONAL CENTRE FOR APPLIED SCIENCES**
**END SEMESTER THEORY EXAMINATION - NOVEMBER/DECEMBER 2023**
**III SEMESTER B.Sc. (APPLIED SCIENCES) IN ENGG.**

**Mathematics - III [IMA 231]**

**Marks: 50** **Duration: 180 mins.**

**Answer all the questions.**

**Missing data, if any, may be suitably assumed**

1) (4)

A) There are three bags. The first bag contains 1 white, 2 red, and 3 green balls; the second bag contains 2 white, 3 red, and 1 green balls, and the third bag contains 3 white, 1 red, and 2 green balls. Two balls are drawn from a bag chosen at random. These are found to be one white and one red. Find the probability that the balls so drawn came from the first bag, second bag, and third bag.

B) (3)

Derive mean and variance of Binomial distribution.

C) (3)

Show that the number of partitions of 'n' in which every part is odd is equal to the number of partitions of 'n' with unequal (or distinct) parts.

2) (4)

A) Suppose that the two-dimensional random variable (X, Y) is uniformly distributed over the triangular region, R = $\{(x,y)|0 < x < y < 1\}$.

(i) Find its pdf.
(ii) Find the marginal pdf of X and Y.
(iii) Find the correlation coefficient between X and Y.

B) (3)

Suppose that an electronic device has a life length X (in 1000 hours) which is considered as a continuous random variable with pdf $f(x) = e^{-x}; x > 0$. Suppose that the cost of manufacturing one such item is 2 dollars and the manufacturer sells the item for 5 dollars, but guarantees for a total refund if $X \leq 0.9$. What is the manufacturer's expected profit per item?

C) (3)

Find the number of permutations of the word 'INSTITUTION'. How many of them begin with 'I' and end with 'N'?

3) (4)

A)

Solve the LPP by Simplex method. Max Z = $5x_1 + 3x_2$
Subject to the constraints,
$x_1 + x_2 \leq 2;$
$5x_1 + 2x_2 \leq 10;$
$3x_1 + 8x_2 \leq 12$

$3x_1 + 6x_2 \leq 12$
And $x_1 \geq 0; x_2 \geq 0.$

B) Suppose that the temperature (in degrees) is normally distributed with expectation $50^{\circ}$ and variance 4. What is the expectation that the temperature will be between $48^{\circ}$ and $53^{\circ}$. (3)

C) Use Chebyshev inequality to find the bounds for $\boldsymbol{P(-1 < X < 7)}$ and $\boldsymbol{P(|X - \mu| \geq 3)}$ if $\mu = 3, E(X^2) = 13$. (3)

4) (4)

A) Find the moment generating function of Gamma distribution and then derive its mean and variance.

B) A company makes two kinds of leather belts. Type 'A' is high quality and type 'B' is low quality. The respective profits are Rs. 4 and Rs. 3 per belt. Each belt of type 'A' requires twice as much time as a belt of type 'B' and if all belts were of the type 'B', that company could make 1000 per day. The supply of leather is sufficient for only 800 belts per day both A and B combined. Belt 'A' requires a fancy buckle and only 400 per day available. There are only 700 buckles available for belt 'B'. Formulate the LPP. (3)

C) Let X have Cauchy's distribution, then prove that $\frac{1}{X}$ also has Cauchy's distribution. (3)

5) (4)

A) Two independent random variable X and Y having pdf
$f(x) = e^{-x}, g(x) = 2e^{-2y}, 0 \leq x \leq \infty; 0 \leq y \leq \infty.$
Find the pdf of (X + Y).

B) Solve the given LPP by graphical method. (3)

Max $Z = 3x_1 + 2x_2$
Subject to the constraints,
$x_1 - x_2 \leq 1;$
$x_1 + x_2 \geq 3$
and $x_1 \geq 0; x_2 \geq 0.$

C) If X is a random variable taking values 0, 1, 2, ..... and $P(X) = ab^x$, where 'a' and 'b' are positive constants such that $a + b = 1$. (3)

**(i)** Find moment generating function.
**(ii)** If $E(X) = m_1, E(X^2) = m_2,$ show that $m_2 = m_1(2m_1 + 1)$.

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