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## DEPARTMENT OF SCIENCES I SEMESTER M.Sc. (PHYSICS) END SEMESTER REGULAR EXAMINATIONS, NOVEMBER & DECEMBER 2023 Quantum Mechanics - I [PHY 5153] (CHOICE BASED CREDIT SYSTEM - 2020)

Time: 3 Hours

Date: 04/12/2023

MAX. MARKS: 50

Note (i) Answer ALL questions

(ii) Draw diagrams, and write equations wherever necessary

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Question

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1A	Describe linear vector space. Give an example.	5	2	2
1B	Interpret expectation value of an operator in quantum mechanics.	3	3	5
1C	What conditions must the parameter $\epsilon$ and the operator $\hat{G}$ satisfy so that the operator $\hat{U} = e^{i\epsilon\hat{G}}$ is unitary?	2	2	4

2A	Explain the basic postulates of quantum mechanics.	5	3	4
2B	Show that			
	$\frac{d}{dt}\langle \hat{x}\rangle = \frac{1}{m}\langle \hat{p}\rangle.$ Discuss the significance of the above equation.	3	3	3
2C	Define time evolution operator. Is the time evolution in quantum mechanics unitary? Justify.	2	3	1, 5

ЗA	Find the bound state wave function and the corresponding energy of a particle in a delta function potential well $V(x) = -\alpha  \delta(x)$ .	4	4	3
3B	The ground state $\psi_0$ of the 1D harmonic oscillator can be defined using the condition $\hat{a}\psi_0 = 0$ . Using the above condition, determine the normalized ground state wave function. Also, find the ground state energy. Given $\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}(i\hat{p} + m\omega x)$	4	4	3
3C	Compare the energies of 3D rectangular potential well and cubic potential well. Which of these systems exhibits degeneracy and explain why?	2	4	4

4A	The angular part of the 3D Schrodinger equation for a spherically symmetric potential is given by $\sin\theta \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial Y}{\partial\theta}\right) + \frac{\partial^2 Y}{\partial\phi^2} = l(l+1)\sin^2\theta Y.$	4	4	3
	Reduce the above equation to two 1-dimensional equations separately for $\theta$ and $\phi$ . Solve the resulting $\phi$ equation.			

4B	Evaluate i) $\begin{bmatrix} L^2 & L_x \end{bmatrix}$ ii) $L_+  l, m\rangle$ , where $ l, m\rangle$ is simultaneous eigenstate of $L^2$ and $L_z$ .	4	5	5
4C	Construct the ground state wave function of the Hydrogen atom. Given: $c_{j+1} = \frac{2 \ (j+l+1-n)}{(j+1)(j+2l+2)} c_j$ and $Y_0^0 = \sqrt{\frac{1}{4\pi}}$	2	4	4

5A	Consider an eigenket $ f\rangle$ of the angular momentum operator $L_z$ . Prove that $L_{\pm} f\rangle$ is also an eigenket of $L_z$ .	5	5	4
5B	Using the matrix representations for the spin operators $S_x$ and $S_y$ , evaluate the commutator $[S_x, S_y]$ .	3	5	5
5C	What are Clebsch–Gordan coefficients? Determine all possible states $ j,m\rangle$ that can be achieved by coupling two angular momenta $j_1 = 1$ and $j_2 = 1/2$ .	2	5	2, 3

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