Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH. (CEM/EM)
END SEMESTER EXAMINATIONS, JANUARY 2023

STATISTICS, PROBABILITY AND RELIABILITY [MAT 5153]

REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 3RD January 2023** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions. Statistical tables may be used.
- Missing data may be suitably assumed.

<table>
<tr><th>Q.NO</th><th>Questions</th><th>Marks</th><th>CO</th><th>BTL</th></tr>
<tr><td>1A.</td><td>Compute the mean deviation from mean and mean deviation from mode for the following distribution. Also find coefficient of dispersion.
<table>
<tr><td>C. I.</td><td>0 - 10</td><td>10 - 20</td><td>20 - 30</td><td>30 - 40</td><td>40 - 50</td><td>50 – 60</td></tr>
<tr><td>f</td><td>3</td><td>7</td><td>15</td><td>12</td><td>8</td><td>5</td></tr>
</table></td><td>4</td><td>1</td><td>5</td></tr>
<tr><td>1B.</td><td>Compute the quartile coefficient of skewness for the following distribution.
<table>
<tr><td>C. I.</td><td>1 - 5</td><td>6 - 10</td><td>11 - 15</td><td>16 - 20</td><td>21 - 25</td><td>26 - 30</td><td>31 - 35</td></tr>
<tr><td>f</td><td>3</td><td>4</td><td>68</td><td>30</td><td>10</td><td>6</td><td>2</td></tr>
</table></td><td>3</td><td>1</td><td>5</td></tr>
<tr><td>1C.</td><td>Ten competitors in a music contest are ranked by three judges as follows:
<table>
<tr><th>Judges/Competitors</th><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>A</td><td>6</td><td>5</td><td>3</td><td>10</td><td>2</td><td>4</td><td>9</td><td>7</td><td>8</td><td>1</td></tr>
<tr><td>B</td><td>5</td><td>8</td><td>4</td><td>7</td><td>10</td><td>2</td><td>1</td><td>6</td><td>9</td><td>3</td></tr>
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</table>
Discuss which pair of judges has the nearest approach to common tastes of music.</td><td>3</td><td>1</td><td>5</td></tr>
<tr><td>2A.</td><td>In an intelligence test administered to 1000 children, the average score is 42 and standard deviation 24.
(i) Find the number of children exceeding the score 60.
(ii) Find the number of children with score lying between 20 and 40. (Assume the normal distribution).</td><td>4</td><td>2</td><td>3</td></tr>
<tr><td>2B.</td><td>The regression lines of x on y and y on x respectively are
$x = 0.854y$ and y=0.89x, $\sigma_x = 3$. Calculate
(i) Coefficient of correlation between x and y.
(ii) Standard deviation of y.</td><td>3</td><td>1</td><td>5</td></tr>
</table>

2C.	If` $\overline{X}$ is mean of a random sample of size n from a distribution N(μ, 100). Then find n such that $P\left(\mu-5<\overline{X}<\mu+5\right)=0.954$.	3	3	3
3A.	Let X have a pdf of the form $f(x,\theta)=\begin{cases}\theta x^{\theta-1}, 0<x<1\\ 0, \quad \text{elsewhere}\end{cases}$, where $\theta \in \{\theta : \theta = 1, 2\}$. To test the simple hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 2$, use a random sample X_1, X_2 of size n = 2 and define the critical region to be $C=\left\{(x_1,x_2): \frac{3}{4} \le x_1x_2\right\}$. Find the power function of the test and the significance level.	4	4	5
3B.	The continuous random variable X has uniform distribution over the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$. Find the pdf of $Y=\tan X$.	3	2	4
3C.	The moment generating function of a random variable X is $e^{9(e^t-1)}$. Find $P(0<X<\mu-2\sigma)$.	3	2	3
4A.	A random sample of size 15 from a population $N(\mu,\sigma^2)$ yields $\overline{X}$ = 3.2 and S^2 = 4.24. Find a 90 percent confidence interval for σ^2.	4	2	5
4B.	Mendelian theory predicts ratio of round and yellow, wrinkled and yellow, round and green, wrinkled and green as 9 : 3 : 3 : 1. From a sample of 160, the actual numbers observed are 86, 35, 26, 13. Is the data consistent with the theory at 0.01 significance level?	3	4	4
4C.	Let $(X_1, X_2, \ldots, X_n)$ denote a random sample of size n from a distribution having pdf $f(x,\theta)=\begin{cases}\frac{\theta^x e^{-\theta}}{x!}, & x=0,1,2,\ldots; \quad 0\le\theta\le 1\\ 0, & \text{elsewhere}\end{cases}$. Find M.L.E for θ.	3	2	3
5A.	Let T, the time to failure be a continuous random variable assuming all non-negative values. Prove that T has an exponential distribution if and only if it has constant failure rate.	4	5	3
5B.	Suppose that T, the time to failure of an item is normally distributed with E(T) = 90 hours and standard deviation 4 hours. In order to achieve a reliability of 0.90, 0.95, 0.99, how many hours of operation may be considered ?	3	5	5
5C	From past experience, it is known that the number of tickets purchased by a student standing in line at the ticket window for the football match follows a distribution that has mean 2.4 and standard deviation 2.0. Suppose that few hours before the start of one of these matches there are 100 eager students standing in line to purchase tickets. If only 250 tickets remain, what is the probability that all 100 students will be able to purchase the tickets they desire?	3	3	3

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