Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH. STRUCTURAL ENGINEERING)
END SEMESTER EXAMINATIONS, JANUARY 2023

OPTIMIZATION TECHNIQUES [MAT 5154]
REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 07 January 2023** **Max. Marks: 50**

Instructions to Candidates:

- Answer **ALL** the questions.
- Missing data may be suitably assumed.

<table>
<tr><th>Q.NO</th><th>Questions</th><th>Marks</th><th>CO</th><th>BTL</th></tr>
<tr><td>1A.</td><td>Use three-point interval search to approximate the location of the global minimum of $xsin4x$ on [0, 3] to within ∈= 0.01.</td><td>3</td><td>CO1</td><td>2</td></tr>
<tr><td>1B.</td><td>Solve the quadratic programming problem
$Max\ Z = 20x_1 - 20x_1^2 + 50x_2 - 50x_2^2 + 18x_1x_2$
subject to $x_1 + x_2 \leq 6, x_1 + 4x_2 \leq 18, x_1, x_2 \geq 0.$</td><td>3</td><td>CO1</td><td>3</td></tr>
<tr><td>1C.</td><td>The manager of an oil refinery has to decide upon the optimal mix of two possible blending process of which the inputs and outputs per production run are as follows:
<table>
<tr><th></th><th colspan="2">Input</th><th colspan="2">Output</th></tr>
<tr><td>Process</td><td>Crude A</td><td>Crude B</td><td>Gasoline X</td><td>Gasoline Y</td></tr>
<tr><td>1</td><td>5</td><td>3</td><td>5</td><td>8</td></tr>
<tr><td>2</td><td>4</td><td>5</td><td>4</td><td>4</td></tr>
</table>
The maximum amounts available of crude A and B are 200 units and 150 units respectively. Market requirements shows that at least 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from Process 1 and Process 2 are Rs. 40 and Rs. 50 respectively. Formulate the problem as a linear programming model and solve graphically.</td><td>4</td><td>CO2</td><td>2</td></tr>
<tr><td>2A.</td><td>Fit a curve of the form $y = ax^b + c$ for the following data, by the method of group averages.
<table>
<tr><td>x</td><td>250</td><td>500</td><td>900</td><td>1200</td><td>1600</td><td>2000</td></tr>
<tr><td>y</td><td>0.25</td><td>0.38</td><td>0.8</td><td>1.38</td><td>2.56</td><td>4.1</td></tr>
</table></td><td>3</td><td>CO4</td><td>2</td></tr>
<tr><td>2B.</td><td>Reduce the matrix $\begin{pmatrix} 4 & 2 & 2 \\ 2 & 5 & 1 \\ 2 & 1 & 6 \end{pmatrix}$ to tridiagonal and find all the eigenvalues of the same.</td><td>3</td><td>CO3</td><td>2</td></tr>
<tr><td>2C.</td><td>Solve the nonlinear programming problem
$Max\ z = 6x_1 + 8x_2 - x_1^2 - x_2^2$ subject to the constraints
$4x_1 + 3x_2 = 16, 3x_1 + 5x_2 = 15,$ with x_1, x_2 non negative.</td><td>4</td><td>CO1</td><td>3</td></tr>
</table>

3A.	Show by simplex method that the linear programming problem $Max\ Z = 5x_1 + 4x_2$ subject to the constraint $x_1 \leq 7, x_1 - x_2 \leq 8, x_1, x_2 \geq 0$ has an unbounded solution.	3	**CO2**	**3**
3B.	Use Fibonacci search method to approximate the global minimum of $x + \frac{4}{x}$ on $(0, 2]$ with in $\in = 0.2$.	3	**CO1**	**2**
3C.	Find the optimal solution to **dual** of $Min\ Z = 3x_1 + 3x_2$ subject to $2x_1 + 4x_2 \geq 40, 3x_1 + 2x_2 \geq 50. x_1, x_2 \geq 0.$	4	**CO2**	**2**
4A.	Find the smallest eigenvalue of matrix by power method. $\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ Carry out four iterations by taking $(1, 1, 1)'$ as the initial vector.	3	**CO3**	**3**
4B.	Solve by two-phase method $Max\ Z = 5x_1 + 3x_2$ subject to the constraint $2x_1 + x_2 \leq 1, x_1 + 4x_2 \geq 6, x_1, x_2 \geq 0.$	3	**CO2**	**3**
4C.	Solve by dual simplex method $Max\ Z = -3x_1 - 2x_2$ subject to the constraints $x_1 + x_2 \geq 1, x_1 + x_2 \leq 7, x_1 + 2x_2 \geq 10, x_2 \leq 3$ with $x_1, x_2 \geq 0.$	*4*	**CO2**	**4**
5A.	Solve by Big-M method $Max\ Z = 5x_1 - 2x_2 + 3x_3$ subject to the constraints $2x_1 + 2x_2 - x_3 \geq 2, 3x_1 - 4x_2 \leq 3, x_2 + 3x_3 \leq 5,$ $x_1, x_2, x_3 \geq 0.$	*5*	**CO1**	**4**
5B.	Solve the LPP by simplex method $Max\ Z = 4x_1 + 10x_2$ subject to $2x_1 + x_2 \leq 10, 2x_1 + 5x_2 \leq 20, 2x_1 + 3x_2 \leq 18, x_1, x_2 \geq 0.$ a. Indicate that this problem has an alternate optimal basic feasible solution and find that optimal solution. b. Hence show that this problem has multiple optimal solutions.	*5*	**CO1**	**4**