MANIPAL ACADEMY of HIGHER EDUCATION (Institution of Eminence Deemed to be University)

Reg.No.									

M.Tech. I SEMESTER (CSE)
END SEMESTER EXAMINATIONS, JANUARY 2023
Computational Methods and Stochastic Processes [MAT 5152]

Time: 3 Hours | Date: 10 01 2023 | MAX. MARKS: 50

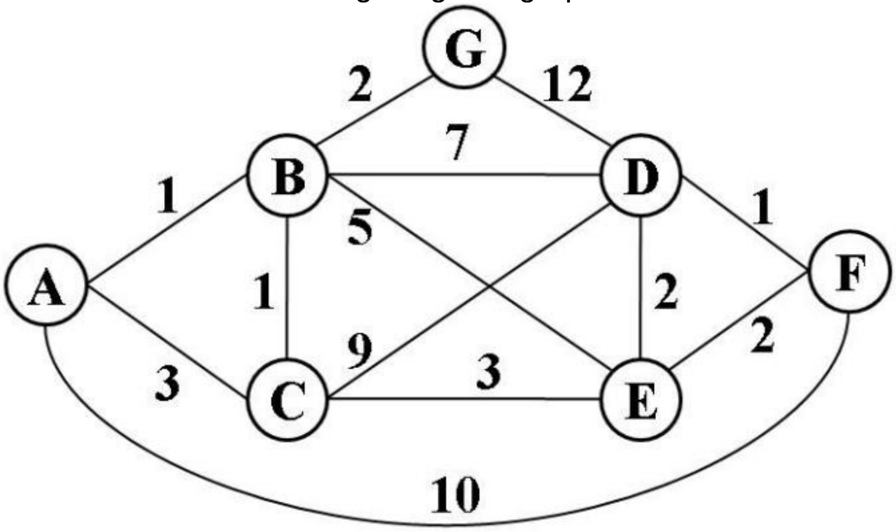
Note (i) Answer ALL questions
(ii) Draw diagrams, and write equations wherever necessary

		Marks	CO	BL
1A	Find the value of the game $\begin{array}{cccccc} & B_1 & B_2 & B_3 & B_4 & B_5 \\ A_1 & -4 & 2 & 5 & -6 & 6 \\ A_2 & 3 & -9 & 7 & 4 & 8 \end{array}$.	3	1	2
1B	Two players A and B will simultaneously place a coin on the table. If the coins `match' (both heads), then player A is paid Rs 8 by player B. If the coins `match' (both tails), then player A is paid Rs 1 by player B. If the coins do not `match', then player B is paid Rs 3 by player A. Given the choice of player A or player B, which one would you choose?	3	1	3
1C	Using Simplex method, solve the LPP $\max z = 3x_1 + 2x_2$ subject to the constraints $2x_1 + x_2 \le 18$, $2x_1 + 3x_2 \le 42$, $3x_1 + x_2 \le 24$ and $x_1 \ge 0, x_2 \ge 0$.	4	1	3

		Marks	CO	BL
2A	The joint probability function for two discrete random variables X and Y is given by $f(x,y) = \begin{Bmatrix} c(2x+y) & 0 \le x \le 2;\ 0 \le y \le 2 \\ 0 & \text{otherwise} \end{Bmatrix}$, where x and y can assume only integral values. Compute the value of c and find the $\mathrm{COV}(X,Y)$.	3	2	3
2B	Let $\bar{X}$ denote the mean of the random sample of size 100 drawn from a distribution with mean 50 and variance 100. Compute $P(49 < \bar{X} < 51)$. It is given that $P(Z \le 1) = 0.8413$, where Z denotes the standard normal variate.	4	2	5
2C	Consider the process $X(t) = A\cos wt + B\sin wt$, where A, B are uncorrelated random variables each with mean 0 and variance 1 and w is a positive constant. Show that the process $X(t)$ is covariance stationary.	3	3	3

		Marks	CO	BL
3A	A student's study habits are as follows: If he studies one night, he is 30% sure to study next night. On the other hand, if he does not study one night, he is 40% sure to study next night. Compute fixed probability vector, to know how often he studies in the long run.	3	3	5
3B	Find the nature of the states in a 3-state process whose one-step transition matrix is $P = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$.	3	3	4
3C	Obtain the singular value decomposition of the matrix $A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}$.	4	4	4

		Marks	CO	BL
4A	Diagonalize the matrix $B = \begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix}$.	4	4	2
4B	Obtain the QR decomposition of the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.	3	4	4
4C	Five defective LED bulbs are accidently gotten mixed with 20 good ones. It is not possible to just look at a bulb and tell whether it is defective or not. Determine the probability distribution of the number of defective bulbs if four bulbs are drawn at random from this lot.	3	2	5

		Marks	CO	BL
5A	Using Dijkstra's algorithm find the shortest path from the node A to every other node in the following weighted graph 	3	5	4
5B	Using finite difference method, solve the elliptic equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ for $0 < x < 1$ and $0 < y < 1$ under the condition $u(x,0) = u(0,y) = 0$, $u(1,y) = 9(y - y^2)$, $u(x,1) = 9(x - x^2)$ by taking $h = 1/2$.	3	6	4
5C	Using finite difference method, solve the hyperbolic equation $\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial y^2}$ taking $h = 1$ and $k = 1/4$ upto $t = 5/4$. The boundary conditions $u(0,t) = u(5,t) = 0$, $u_t(x,0) = 0$, and $u(x,0) = x^2(5 - x)$.	4	6	4

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