

I SEMESTER M. TECH (Computer Science & Engineering) END SEMESTER EXAMINATION, December 5, 2023 SUBJECT: QUANTUM COMPUTING (CSE 5115) REVISED CREDIT SYSTEM

Time: 3 Hours (9.30 AM-12.30 AM)

Note: Answer ALL the questions.

MAX. MARKS: 50

1A	A qubit is prepared in state $ \psi\rangle = \frac{1}{\sqrt{3}} 0\rangle - \sqrt{\frac{2}{3}} 1\rangle$. It is then measured. Compute P(1)? The qubit is measured a second time. Calculate P(0) and P(1) for the second measurement, given that the first measurement gave result 1?	3
1B	Define "braket" of two vectors $ v\rangle$ and $ w\rangle$. If $ v\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1\\i \end{bmatrix}$ and $ w\rangle = \frac{i}{\sqrt{2}} \begin{bmatrix} 1\\1 \end{bmatrix}$, compute $\langle v w\rangle$.	3
1C	Determine whether the following two qubit state is entangled or not. Justify your answer. $\frac{ 00\rangle + 01\rangle}{\sqrt{2}}$	4
2A	Suppose you are given a system with a two qubit function $ \psi\rangle = \frac{1}{2} (00\rangle + 01\rangle + 10\rangle + 11\rangle)$. Can this state be expressed as a separable state? That is, can you find single qubit functions $ \varphi_1\rangle$ and $ \varphi_2\rangle$ such that $ \psi\rangle = \varphi_1\rangle \otimes \varphi_2\rangle$? If yes, please find an expression for $ \varphi_1\rangle$ and $ \varphi_1\rangle$. If no, explain why not.	4
2B	Let H and I are Hadamard and Identity gates respectively and $ \psi\rangle$ is as defined in Q2A. Compute $ \psi_1\rangle = (H \otimes I) \psi\rangle$	3
2C	Define CNOT gate. Design CNOT circuit and compute CNOT matrix.	3
3A	Define I Gate. Derive the Braket representation of I gate.	3
3B	Show that SWAP can be implemented using CNOT gates. Design the corresponding SWAP circuit. Compute SWAP matrix.	3
3C	Explain No Cloning Principle. Illustrate No cloning principle using CNOT Gate.	4
4A	Compute the output of the following quantum circuit. $ 0\rangle - H - H - H - H - H - H - H - H - H - $	4
4B	Alice wishes to send Bob a message via a secure protocol. She chooses to use a private key encryption technique and decides to use the BB84 protocol to generate a provably secure private encryption key. Alice's first step is to generate a random binary string. The string she generates is $b = 010101110110000$. Alice then encodes this as a string of quantum qubits as per the BB84 protocol, using the encoding H H H I I H H H I I H H I I H. What is the quantum string that she	4

	generates? Alice then sends the string to Bob, who decodes using I H H I H I H I H I H I H I H I H I H	
4C	Suppose that charlie intercepts the qubit transmitted by Alice in the superdense coding protocol. Can she infer which of the four pairs of bits 00; 01; 10, or 11 Alice was trying to transmit? If so, how? If not, why not?	2
5A	Apply Grover's quantum search algorithm on a system with $N=4$ and solution is indexed by $x = 0$.	5
5B	Let H is Hadamard gate, compute $H^{\otimes n} 0\rangle^{\otimes n}$ and express result in summation form.	3
5C	Let $ \psi\rangle = a 0\rangle + b 1\rangle$ be a quantum state, Alice wants to send to Bob. To reduce channel errors, Alice encode 0 by 000 and 1 by 111. Design quantum circuit to implement this encoding.	2