Reg. No.									

MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL

(A constituent unit of MAHE, Manipal)

FIRST SEMESTER M.TECH. (DEAC/ME ENGINEERING)
END SEMESTER EXAMINATIONS, JANUARY 2023

PROBABILITY, RANDOM VARIABLES AND STOCHASTIC PROCESSES [MAT 5151]
REVISED CREDIT SYSTEM

Time: 3 Hours **Date: 07 January 2023** **Max. Marks: 50**

Instructions to Candidates:
- Answer **ALL** the questions.
- Missing data may be suitably assumed.

Q.NO	Questions	*Marks*	*CO*	*BTL*
1A.	A certain type of electric motor fails either by seizure of the bearings or by burning out of the electric windings or by wearing out of the brushes. Suppose that seizure is twice as likely as burning out which is four times as likely as brush wear out, what is the probability that failure will be by each of these three mechanisms?	3	1	evaluate
1B.	A computer system is built so that if component K_1 fails, it is bypassed and K_2 is used. If K_2 fails, then K_3 is used. Suppose the probability that K_1 fails is 0.01, that K_2 fails is 0.03 and that K_3 fails is 0.08, find the probability of the failure of the system, assuming that the failures are mutually independent events.	3	1	evaluate
1C.	In an industry, machines A, B and C manufacture 25, 35 and 40 percent of the total output respectively. Of their outputs, 6, 3 and 1 percent respectively are defective bolts. A bolt is chosen at random and found to be defective. What is the probability that it came from machine C?	4	1	evaluate
2A.	The cumulative distribution function that a television tube will fail in t hours is $1 - e^{-ct}$, where c is a parameter dependent on the manufacturer, and $t \geq 0$. Find the probability density function of the life T of the tube and verify whether it is a valid probability density function.	3	2	Evaluate & analyze
2B.	Fluorescent lamps produced by a manufacturer have a probability of 0.05 of being inoperable when new. A person purchases eight of the lamps for home use. What is the probability that all eight lamps are operable? Determine the probability that exactly one lamp is inoperable	3	2	evaluate
2C.	A spacecraft has 20000 components. The probability of any one component being defective is 10^{-4}. The mission will be in danger if five or more components become defective. Find the probability of such an event.	4	2	evaluate
3A.	The process of drilling holes in printed circuit boards produces diameters with a standard deviation of 0.01 milli meter. How many diameters must be measured so that the probability is at least 8/9 that the average of the measured diameters is within 0.005 of the process mean diameter μ?	3	3	apply
3B.	Assume that the time of arrival of birds at a particular place on a migratory route, as measured in days from the first day *i. e.,* 1st January of the year, is approximated as a Gaussian (normal) random variable X with mean $\mu_X = 200 days$ and standard deviation $\sigma_X = 20 days$. What is the probability that the birds arrive after 160 days but on or before 210th day? What is the probability that the birds will arrive after the 231st day?	3	3	evaluate

<table>
<tr><td></td><td>P(Z<0.2)=0.9772, P(Z<0.5)=0.6915, P(Z<1.55)=0.9394</td><td></td><td></td><td></td></tr>
<tr><td>3C.</td><td>Suppose (X, Y) is uniformly distributed over the semicircle $x^2 + y^2 = 1$ above the initial line having joint probability density function $f(x, y) = \begin{cases} \frac{2}{\pi} \forall (x, y) above\ initial\ line \\ 0, otherwise \end{cases}$. Find the correlation coefficient between the random variables X and Y.</td><td>4</td><td>3</td><td>analyze</td></tr>
<tr><td>4A.</td><td>The demand for a particular spare part in a factory was found to vary from day to day. In a sample study, the following information regarding the demand was obtained. Test the hypothesis that the number of parts demanded does not depend on the day of the week. Table value or threshold at 5% level of significance is 11.1
<table>
<tr><td>Day</td><td>Monday</td><td>Tuesday</td><td>Wednesday</td><td>Thursday</td><td>Friday</td><td>Saturday</td></tr>
<tr><td>No. of parts demanded</td><td>1124</td><td>1125</td><td>1110</td><td>1120</td><td>1126</td><td>1115</td></tr>
</table></td><td>3</td><td>4</td><td>analyze</td></tr>
<tr><td>4B.</td><td>The life length in months of a random sample of ten bulbs are 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean life of 100 months? Test this at 5% level of significance. The threshold value is 2.26. Also find a 95% confidence interval for the mean of the population.</td><td>3</td><td>4</td><td>Analyze & evaluate</td></tr>
<tr><td>4C.</td><td>A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single factor experiment was conducted with three dosage levels and the following results were obtained.
<table>
<tr><td>Dosage</td><td>Observations</td><td></td><td></td><td></td></tr>
<tr><td>20g</td><td>24</td><td>28</td><td>37</td><td>30</td></tr>
<tr><td>30g</td><td>37</td><td>44</td><td>39</td><td>35</td></tr>
<tr><td>40g</td><td>42</td><td>47</td><td>52</td><td>38</td></tr>
</table>
Is there evidence to indicate that dosage level affects bioactivity? The threshold at 5% level of significance is 4.26.</td><td>4</td><td>4</td><td>Analyze</td></tr>
<tr><td>5A.</td><td>A prisoner in a prison is put in the following situation. A regular deck of 52 cards is placed in front of him. He must choose cards one at a time to determine their color. Once chosen, the card is replaced in the deck and the deck is shuffled. If the prisoner happens to select three consecutive red cards, he is executed. If he happens to select six cards before three consecutive red cards appear, he is granted freedom. Construct the transition probability matrix after representing the prisoner’s situation as a Markov chain.</td><td>3</td><td>5</td><td>Analyze & create</td></tr>
<tr><td>5B.</td><td>Check for stationarity in the weak sense for the process $X(t) = A\ cos\omega t + Bsin\omega t$, where A and B are uncorrelated random variables each with mean 0 and variance 1 and ω is a positive constant.</td><td>3</td><td>5</td><td>Analyze</td></tr>
<tr><td>5C.</td><td>A graduate research assistant moonlights in the food court in the student union in the evenings. He is the only one on dutyat the counter during the hours he works. Arrivals to the counter seem to follow the Poisson distribution with mean of 10/hour. Each customer is served one at a time and the service time is thought to follow an exponential distribution with a mean of 4 minutes. What is the probability of having a queue? What is the average queue length? What is the average time a customer spends in the system?</td><td>4</td><td>5</td><td>Analyze & evaluate</td></tr>
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