Question Paper

Exam Date & Time: 30-Apr-2024 (09:30 AM - 12:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

SECOND SEMESTER B.TECH. EXAMINATIONS - APRIL / MAY 2024
SUBJECT: MAT 1271/MAT_1271 - ENGINEERING MATHEMATICS - II
(CHEMISTRY GROUP)

Marks: 50 **Duration: 180 mins.**

Answer all the questions.

1A) Determine the minimum value of x^2yz^3 subjected to the condition $2x + y + 3z = 3$. (4)

1B) If $z = 4x - y^2$ where $x = uv^2$ and $y = u^3v$ then find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$. (3)

1C) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$. (3)

2A) Show that the plane $x + 2y - z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x - z - 2 = 0$ in a circle of radius unity. Furthermore, find the equation of the sphere which has this circle as a great circle. (4)

2B) Expand $f(x, y) = e^x log(1 + y)$ in powers of x and y up to the 3rd degree terms. (3)

2C) Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$. (3)

3A) Test the convergence of the series (4)

$$\frac{2^2}{3^2} + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2} + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2} + \cdots$$

3B) Evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dy\, dx$ by changing to polar coordinates. (3)

3C) Use double integration to find the area lying inside the cardioid $r = 2(1 + cos\theta)$ and outside the circle $r = 2$. (3)

4A) Using Laplace transforms, solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 1 - e^{2t}$ subject to the conditions $y(0) = 1$ and $y'(0) = 0$. (4)

4B) Evaluate $L^{-1}\left(\frac{2s+1}{(s-1)^2(s+2)^2}\right)$. (3)

4C) Express the function $f(t) = \begin{cases} t-1; & 1<t<2 \\ 3-t & ;2<t<3 \end{cases}$ in terms of unit step function and hence find $L\{f(t)\}$. (3)

5A) Test the absolute and conditional convergence of the following series (4)

$$x - \frac{x^2}{\sqrt{3}} + \frac{x^3}{\sqrt{5}} - \cdots$$

5B) Using triple integrals, find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and $z = 0$. (3)

5C) Use beta and gamma functions to show that: (3)

$$\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \sqrt{\cot\theta}\, d\theta = \frac{\pi^2}{2}$$

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