Question Paper

MANIPAL ACADEMY OF HIGHER EDUCATION

SECOND SEMESTER B.TECH. DEGREE EXAMINATIONS - APRIL/MAY 2024
SUBJECT: MAT 1271/MAT_1271 - ENGINEERING MATHEMATICS - II
(PHYSICS GROUP)

Marks: 50   **Duration: 180 mins.**

Answer all the questions.

1A) Find the maxima and minima of the function   (5)

$$f(x, y) = x^2 + 2xy + 2y^2 + 2x + y.$$

1B)   (5)

(i)   Using Euler's theorem, if $u = \log\left(\frac{x^3+y^3}{2x+3y}\right)$ then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2.$$

(ii)   If $u = x^3 + y^3 - 3x^2y + 4$ then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

2A) Find the equation of the sphere having the circle   (4)

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0; \quad x + y + z = 3$$

as a great circle.

2B) Expand $f(x, y) = x^2y + 3y - 4$ about the point $(1, -2)$ up to second degree terms.   (3)

2C) Evaluate $\quad \lim\limits_{x\to 0} \dfrac{x^2 + 2\cos x - 2}{x^4}$ (3)

3A) Evaluate   (5)

$$\int_{x=0}^{3}\int_{y=0}^{2}(x^2 + y^2)\,dy\,dx$$

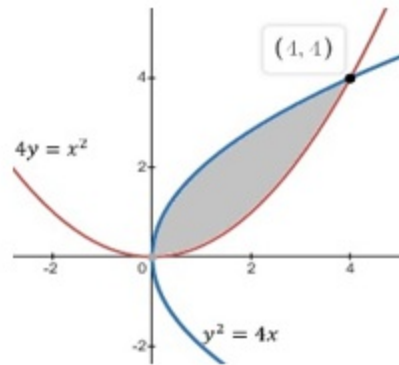**3B)** Using Beta and Gamma functions, evaluate   (5)

$$\int_{0}^{\frac{\pi}{2}}\sqrt{\tan\theta}\,d\theta$$

4A) Using Laplace transforms, solve $y'' + 5y' + 6y = 0$ where $y(0) = 1, y'(0) = 0$.   (5)

4B) Evaluate $L^{-1}\left\{\dfrac{1}{(s+5)(s-9)}\right\}$ (5)

5A) Using double integrals, find the area of the shaded region bounded by the parabolas   (4)

$4y = x^2$ and $y^2 = 4x$.

5B)   Use Quotient test and discuss the convergence of the series   (3)

$$\sum_{n=1}^{\infty} \frac{2n-1}{n(n+1)}$$

5C)   Use Ratio test and discuss the convergence of the series   (3)

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

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