MANIPAL INSTITUTE OF TECHNOLOGY

MANIPAL
(A constituent unit of MAHE, Manipal)

DEPARTMENT OF MATHEMATICS
IVSEMESTER B.TECH.(AUTOMOBILE ENGINEERING)
END SEMESTER EXAMINATION
ENFGINEERING MATHEMATICS-IV- MAT 2222

Date of Examination: 14-06-2024 **Time: 2:30- 5:30 PM** **MAX.MARKS: 50**

<table>
<tr><th>Q. NO.</th><th>Question</th><th>M</th><th>CO</th><th>PO</th><th>BL</th></tr>
<tr><td>1A</td><td>From 8 positive and 6 negative integers, 4 are chosen at random and are multiplied. i) What is the probability that the product is positive? ii) What is the probability that the product is negative.</td><td>3</td><td>5</td><td>2</td><td>3</td></tr>
<tr><td>1B</td><td>Two digits are selected at random from digits 1 to 4.
a) If the sum is even, what is the probability that 3 is one of the digits selected.
b) If 3 is one of the digits selected., what is the probability of the sum is even.</td><td>3</td><td>3</td><td>2</td><td>3</td></tr>
<tr><td>1C</td><td>Given the following table
<table><tr><td>(x, y)</td><td>(1, 1)</td><td>(1, 2)</td><td>(1, 3)</td><td>(2, 1)</td><td>(2, 2)</td><td>(2, 3)</td></tr><tr><td>p(x, y)</td><td>2/15</td><td>4/15</td><td>3/15</td><td>1/15</td><td>1/15</td><td>4/15</td></tr></table>a) Form the Joint probability distribution function.
b) Marginal pmf of X and Y
c) Conditional pmf of X given Y=2 and given X=1
d) Examine whether X and Y are independent.</td><td>4</td><td>4</td><td>2</td><td>3</td></tr>
<tr><td>2A</td><td>2% of the population has a certain blood disease in serious form, 10% have it in a mild form and 88% do not have it at all. A new blood test is developed. The prob. that test is positive is 0.9 if the subject has the serious form; 0.6 if the subject has the mild form and 0.1 if the subject does not have the disease. A subject has tested positive. What is the probability that the subject has the serious form of the disease?</td><td>3</td><td>3</td><td>1, 2</td><td>4</td></tr>
<tr><td>2B</td><td>A box contains 12 items out of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X. Also determine the Mean and Standard deviation of X.</td><td>3</td><td>3</td><td>2</td><td>4</td></tr>
<tr><td>2C</td><td>Calculate the Standard Deviation, if the random variable X has a pdf
$f(x) = \begin{cases} \frac{3+2x}{18}, & 2 \le x \le 4 \\ 0 & , \text{ otherwise} \end{cases}$</td><td>4</td><td>4</td><td>2</td><td>4</td></tr>
</table>

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3A	Find the correlation coefficient for the data given below x: 1, 2, 3, 4, 5 y: 2, 5, 3, 8, 7	3	2	3	3
3B	Suppose X is uniformly distributed over (-1, 1). Find E(Y) where $Y = 4 - X^2$	3	5	2	3
3C	Fit a straight line $y = a + bx$ by the method of least square for the data given below x: 1, 3, 4, 6, 8, 9, 11, 14 y: 1, 2, 4, 4, 5, 7, 8, 9	4	2	2	3
4A	Using Rodrigue's formula, express $f(x) = 5x^3 - 3x^2 - 3x - 6$ in terms of Legendre's polynomial.	3	1	3	4
4B	Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\, cosx$	3	1	3	3
4C	Starting from Jacobi series Prove that $J_0^2 + 2J_1^2 + 2J_2^2 + 2J_3^2 + \ldots = 1$	4	1	3	3
5A	Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer.	3	1	3	3
5B	In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hrs & S. D of 60 hours. If a firm purchases 2500 bulbs. Find the number of bulbs that are likely to last for i) more than 2100 hrs ii) less than1950 hrs iii) between 1900 to 2100 hrs.	3	5	3	4
5C	The joint density function of two random variables X and Y is given $f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \le x \le 1, \quad 1 \le y \le 2 \\ 0, & otherwise \end{cases}$ a) Check f(x, y) is a valid pdf or not b) Find the marginal pdf of X and Y c) Find conditional pdf of X and Y d) Find P(X+Y) <1	4	4	1,2	4