



MANIPAL INSTITUTE OF TECHNOLOGY

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(A constituent unit of MAHE, Manipal)

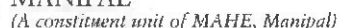
DEPARTMENT OF MATHEMATICS IV SEMESTER B.TECH.(AUTOMOBILE ENGINEERING) END SEMESTER EXAMINATION ENFGINEERING MATHEMATICS-IV- MAT 2222

Date of Examination: 14-06-2024

Time: 2:30- 5:30 PM

MAX.MARKS: 50

Q. NO.	Question	M	CO	PO	BL														
1A	From 8 positive and 6 negative integers, 4 are chosen at random and are multiplied. i) What is the probability that the product is positive? ii) What is the probability that the product is negative.	3	5	2	3														
1B	Two digits are selected at random from digits 1 to 4. a) If the sum is even, what is the probability that 3 is one of the digits selected. b) If 3 is one of the digits selected., what is the probability of the sum is even.	3	3	2	3														
1C	Given the following table <table border="1"><tr><td>(x, y)</td><td>(1, 1)</td><td>(1, 2)</td><td>(1, 3)</td><td>(2, 1)</td><td>(2, 2)</td><td>(2, 3)</td></tr><tr><td>p(x, y)</td><td>2/15</td><td>4/15</td><td>3/15</td><td>1/15</td><td>1/15</td><td>4/15</td></tr></table> a) Form the Joint probability distribution function. b) Marginal pmf of X and Y c) Conditional pmf of X given Y=2 and given X=1 d) Examine whether X and Y are independent.	(x, y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)	p(x, y)	2/15	4/15	3/15	1/15	1/15	4/15	4	4	2	3
(x, y)	(1, 1)	(1, 2)	(1, 3)	(2, 1)	(2, 2)	(2, 3)													
p(x, y)	2/15	4/15	3/15	1/15	1/15	4/15													
2A	2% of the population has a certain blood disease in serious form, 10% have it in a mild form and 88% do not have it at all. A new blood test is developed. The prob. that test is positive is 0.9 if the subject has the serious form; 0.6 if the subject has the mild form and 0.1 if the subject does not have the disease. A subject has tested positive. What is the probability that the subject has the serious form of the disease?	3	3	1, 2	4														
2B	A box contains 12 items out of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X. Also determine the Mean and Standard deviation of X.	3	3	2	4														
2C	Calculate the Standard Deviation, if the random variable X has a pdf $f(x) = \begin{cases} \frac{3+2x}{18}, & 2 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$	4	4	2	4														



3A	Find the correlation coefficient for the data given below	3	2	3	3																		
	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>y</td><td>2</td><td>5</td><td>3</td><td>8</td><td>7</td></tr></table>	x	1	2	3	4	5	y	2	5	3	8	7										
x	1	2	3	4	5																		
y	2	5	3	8	7																		
3B	Suppose X is uniformly distributed over (-1, 1). Find E(Y) where $Y = 4 - X^2$	3	5	2	3																		
3C	Fit a straight line $y = a + bx$ by the method of least square for the data given below	4	2	2	3																		
	<table><tr><td>x</td><td>1</td><td>3</td><td>4</td><td>6</td><td>8</td><td>9</td><td>11</td><td>14</td></tr><tr><td>y</td><td>1</td><td>2</td><td>4</td><td>4</td><td>5</td><td>7</td><td>8</td><td>9</td></tr></table>	x	1	3	4	6	8	9	11	14	y	1	2	4	4	5	7	8	9				
x	1	3	4	6	8	9	11	14															
y	1	2	4	4	5	7	8	9															
4A	Using Rodrigue's formula, express $f(x) = 5x^3 - 3x^2 - 3x - 6$ in terms of Legendre's polynomial.	3	1	3	4																		
4B	Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$	3	1	3	3																		
4C	Starting from Jacobi series Prove that $J_0^2 + 2J_1^2 + 2J_2^2 + 2J_3^2 + \dots = 1$	4	1	3	3																		
5A	Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer.	3	1	3	3																		
5B	In a test on electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hrs & S. D of 60 hours. If a firm purchases 2500 bulbs. Find the number of bulbs that are likely to last for i) more than 2100 hrs ii) less than 1950 hrs iii) between 1900 to 2100 hrs.	3	5	3	4																		
5C	The joint density function of two random variables X and Y is given $f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, \quad 1 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$ a) Check f(x, y) is a valid pdf or not b) Find the marginal pdf of X and Y c) Find conditional pdf of X and Y d) Find $P(X+Y) < 1$	4	4	1,2	4																		