MANIPAL INSTITUTE OF TECHNOLOGY
MANIPAL
(A constituent unit of MAHE, Manipal)

DEPARTMENT OF MATHEMATICS
IVSEMESTER B.TECH.(AUTOMOBILE ENGINEERING)
END SEMESTER EXAMINATION
ENFGINEERING MATHEMATICS-IV- MAT 2222

Date of Examination: 30/04/2024 Time: 2:30 PM – 5:30 PM MAX.MARKS: 50

Q. NO.	Question	M	CO	PO	BL
1A	In a factory turning out razer blades, there is a small probability of 0.002 for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing i) No defective ii) One defective iii) two defective blades in consignment of 10000 packets.	3	5	2	3
1B	Box-I contains 5 Red and 4 Blue marbles, while box-II contains 4 Red and 5 Blue marbles. A fair die is rolled if 1 or 3 turns up, a marble is chosen from Box-I, otherwise a marble is chosen from Box-II. i) Find the probability that a red marble is chosen. ii) If it is given that the chosen marble is red. What is the probability that Box-I is chosen.	3	3	2	3
1C	A fair coin is tossed three times the two random variables X and Y defined as follows, X = 0 or 1 according as head or tail occurs on the first toss. Y denote the total number of heads. Determine the following i) Marginal distributions of X and Y ii) Joint distributions of X and Y iii) E(X) and E(Y) iv) Covariance and correlation of X and Y	4	4	2	3
2A	The chances of A, B, C becoming the general manager of a certain company are in the ratio 4:2:3. The probabilities that the bonus scheme will be introduced in the company if A, B, C become general managers, are 0.3, 0.7and 0.8 respectively. If the bonus scheme has been introduced, what is the probability that A has been appointed as the general manager?	3	3	1, 2	4
2B	A box contains 12 items out of which 4 are defective. A sample of 3 items is selected from the box. Let X denote the number of defective items in the sample. Find the probability distribution of X . Also determine the Mean and Standard deviation of X.	3	3	2	4
2C	The diameter of an electric cable say X is assumed to be continuous random variable with probability density function $f(x) = 6x\,(1-x),\ 0 \le x \le 1$. Find the following i) The distribution function F (x) ii) $P\left[\frac{1}{3} < x < \frac{2}{3}\right]$ iii) $P\left[\frac{x \le \frac{1}{2}}{\frac{1}{3} < x < \frac{2}{3}}\right]$ iv) Find b such that $P(x < b) = 2P(x > b)$	4	4	2	4

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<table>
<tr><td>3A</td><td>Obtain the equations of lines of regression and hence find the coefficient of correlation for the data given below
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>y</td><td>9</td><td>8</td><td>10</td><td>12</td><td>11</td><td>13</td><td>14</td></tr></table></td><td>3</td><td>2</td><td>3</td><td>3</td></tr>
<tr><td>3B</td><td>Derive Mean and Standard deviation of Binomial distributions</td><td>3</td><td>5</td><td>2</td><td>3</td></tr>
<tr><td>3C</td><td>Fit a parabola $y = a + bx + cx^2$ by the method of least square for the data given below
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>y</td><td>2</td><td>6</td><td>7</td><td>8</td><td>10</td><td>11</td><td>11</td><td>10</td><td>9</td></tr></table></td><td>4</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>4A</td><td>Prove that the orthogonality property of Legendre's polynomials in the form of $\int_{-1}^{1} P_m(x)\, P_n(x) dx = \begin{cases} 0, & if\ m \neq n \\ \frac{2}{2n+1}, & if\ m = n \end{cases}$</td><td>3</td><td>1</td><td>3</td><td>4</td></tr>
<tr><td>4B</td><td>Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}}\, sinx$</td><td>3</td><td>1</td><td>3</td><td>3</td></tr>
<tr><td>4C</td><td>i) Express $f(x) = 4x^3 - 2x^2 - 3x - 8$ in terms of Legendre's polynomial
ii) Starting from Jacobi series
Prove that $J_0{}^2 + 2\,J_1{}^2 + 2J_2{}^2 + 2J_3{}^2 + ... = 1$</td><td>4</td><td>1</td><td>3</td><td>3</td></tr>
<tr><td>5A</td><td>Prove that recurrence relation $2n\,J_n(x) = x\,[\,J_{n+1}(x) + J_{n-1}(x)\,]$</td><td>3</td><td>1</td><td>3</td><td>3</td></tr>
<tr><td>5B</td><td>The marks of 1000 students in an examination follows a normal distribution with mean 70 and standard deviation 5. Find the number of students whose marks will be i) Less than 65 ii) More than 75 iii) Between 65 and 75</td><td>3</td><td>5</td><td>3</td><td>4</td></tr>
<tr><td>5C</td><td>The joint density function of two random variables X and Y is given
$f(x, y) = \begin{cases} \frac{1}{8}(6 - x - y), & 0 < x < 2, \quad 2 < y < 4 \\ 0, & \text{otherwise} \end{cases}$
Find: i) E(X), ii) P(X <1, Y<3), iii) P(X+Y) < 3,</td><td>4</td><td>4</td><td>1,2</td><td>4</td></tr>
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