Question Paper

Exam Date & Time: 14-Jun-2024 (02:30 PM - 05:30 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION

IV Semester MakeUp End Semester Examination

ENGINEERING MATHEMATICS - IV [MAT 2223]

Marks: 50 **Duration: 180 mins.**

Descriptive Questions

Answer all the questions. Section Duration: 180 mins

1) A) A bag contains 3 coins, one of which is two headed, while the other 2 coins are normal and not biased. A coin is chosen at random from the bag and tossed 4 times in succession. If head turns up each time, what is the probability that it is a two headed coin? (3)

B) Show that for the normal distribution with mean μ and variance σ^2, (3)

$$E(X-\mu)^{2n} = \sigma^{2n}(1.3.5\ldots(2n-1)).$$

C) Suppose the two-dimensional random variable (X, Y) has the joint probability density function given by (4)

$$f(x,y) = \begin{cases} k(1-x-y), & x>0,\ y>0,\ x+y<1 \\ 0, & otherwise \end{cases}$$

i. Find the value of k.

ii. Determine the marginal density function of X and Y.

2) A) If X_1, X_2, X_3 are uncorrelated random variables having same standard deviation, find the correlation coefficient between X_1+X_2 and X_2+X_3. (3)

B) The probability distribution function of a random variable X is given by (3)

x	0	1	2	3	4	5	6
P(x)	k	3k	5k	7k	9k	11k	13k

Find (i) a. $P(X<4)$ b. $P(X \geq 5)$ c. $P(3 < X \leq 6)$

(ii) What is the minimum value of k, so that $P(X \leq 2) > 0.3$.

C) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution. (4)

3) (3)

A) Let X be a random variable with probability distribution $f(x) = \dfrac{1}{\pi(1+x^2)}$ for $-\infty < x < \infty$. Then find the pdf of $Y = \dfrac{1}{X}$..

B) Suppose that the two-dimensional random variable (X, Y) is uniformly distributed over the triangular region $R = \{(x, y) \mid 0 < x < y < 1\}$. Find its pdf and marginal pdf of X and Y. (3)

C) There are 2 white marbles in box A and 3 red marbles in box B. At each step of the process a marble is selected from each box and the two marbles selected are interchanged. Let the state a_i of the system be the number i of red marbles in box A. (4)

i. Find the transition matrix.

ii. What is the probability that there are 2 red marbles in box A after 3 steps.

4) A) Solve by graphical method Minimize $Z = 40x_1 + 30x_2$ subject to (3)

$$200x_1 + 100x_2 \geq 4000,\ x_1 + 2x_2 \geq 50,\ 40x_1 + 40x_2 \geq 1400,\ \ x_1, x_2 \geq 0.$$

B) Iterate three steps for minimum of $f(x) = 5x_1^2 + x_2^2$ starting from $X_0 = (1,2)$ using steepest descent method. (3)

C) Find the solution using Simplex method (4)

Maximize $Z = 5x_1 + 3x_2$ subject to

$$x_1 + x_2 \leq 2,\quad 5x_1 + 2x_2 \leq 10,\quad 3x_1 + 8x_2 \leq 12,\quad x_1, x_2 \geq 0.$$

5) A) Find the value of constant $(a + b + c)$ so that the directional derivative of the function $f = axy^2 + byz + cz^2x^3$ at the point $(1, 2, -1)$ has maximum magnitude 64 in the direction parallel to y- axis. (3)

B) Show that $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$. (3)

C) A vector field is given by $\vec{A} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$. Show that the field is irrotational and find the scalar potential. (4)

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